A Commutative Diagram of the Heavens

Shotaro Makisumi

Stanford University

Math Day at the Beach, 2014

Introduction

Question

Why are the days of the week in the following order?

Monday Tuesday Wednesday Thursday Friday Saturday Sunday

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- Inspired in part by conversations with Brian Lawrence (Stanford)

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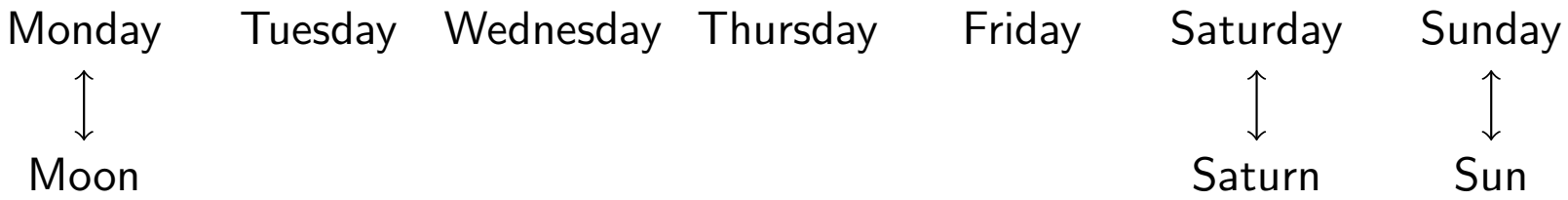

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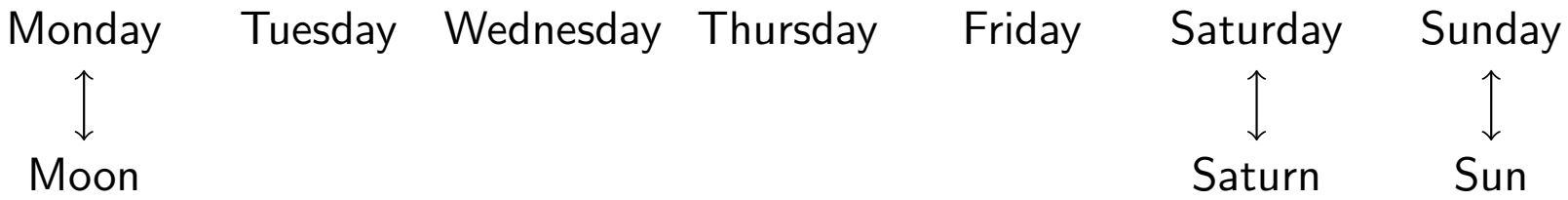

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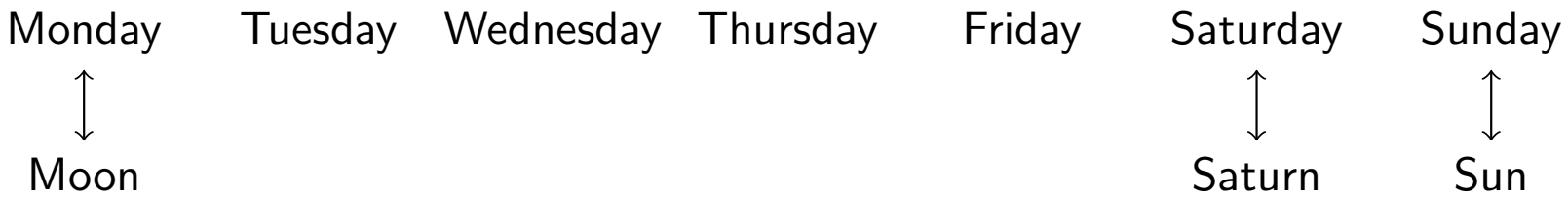

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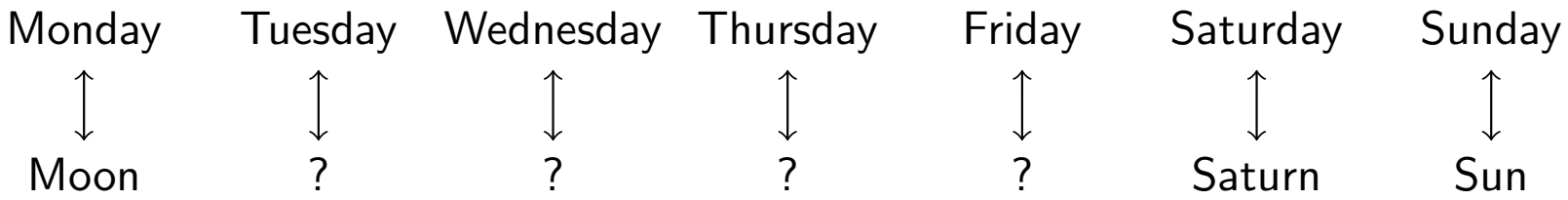

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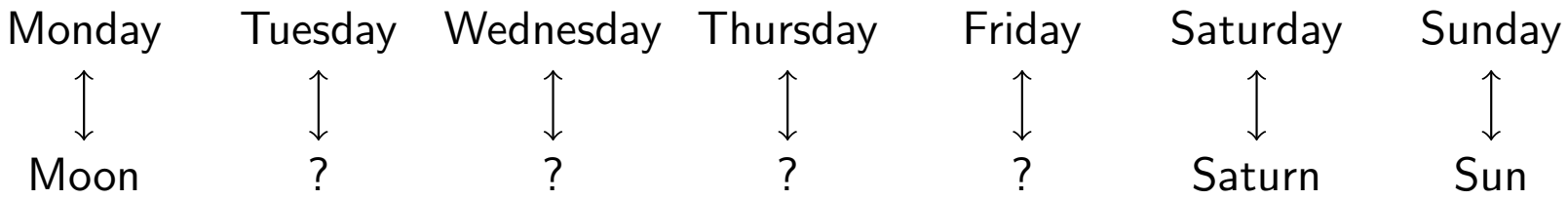

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Goal 2

The order of the days comes from some natural order on the luminaries

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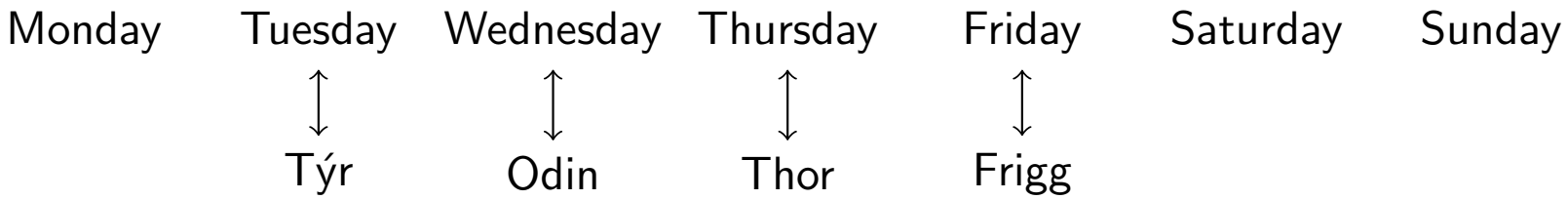

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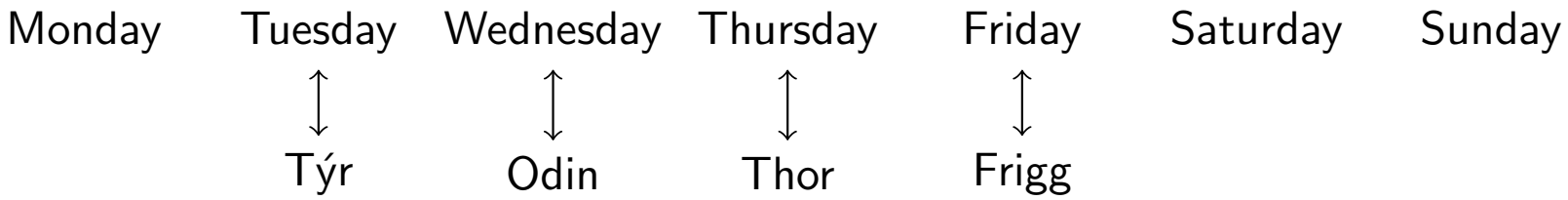

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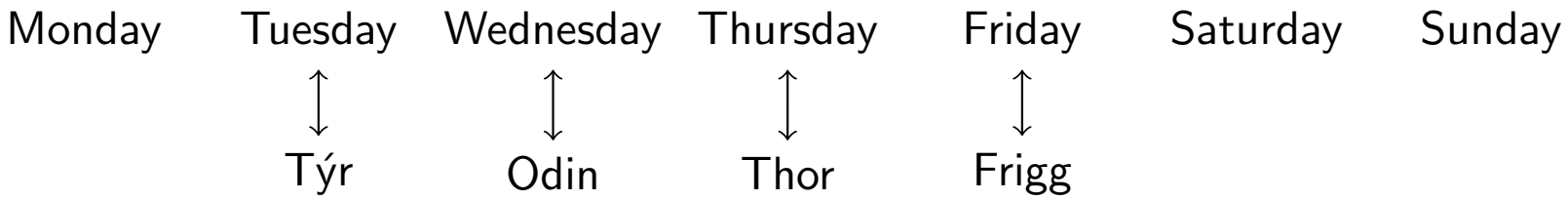

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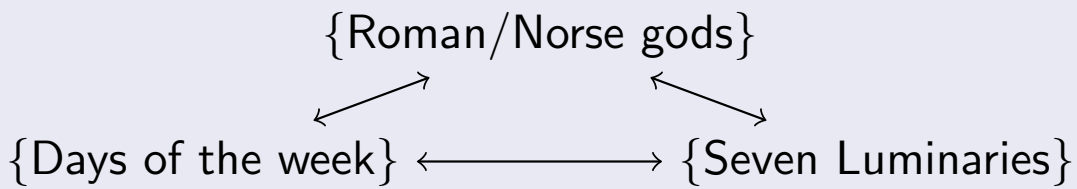

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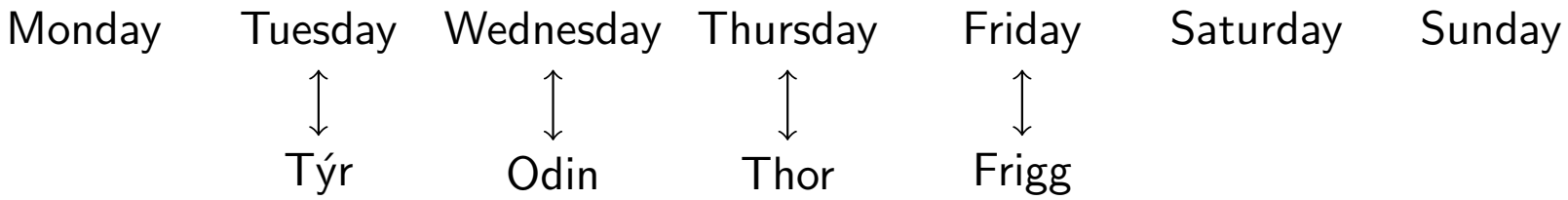

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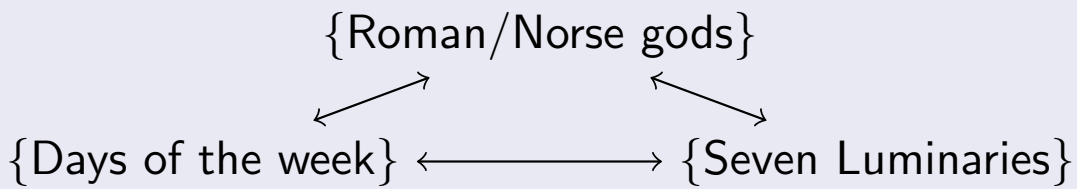

- Remark: Gods don't have a natural order

Goal 1: Days and Luminaries

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- Probably not a coincidence; part of a *single* system of **planetary days**

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So we can construct a correspondence between the days and the luminaries

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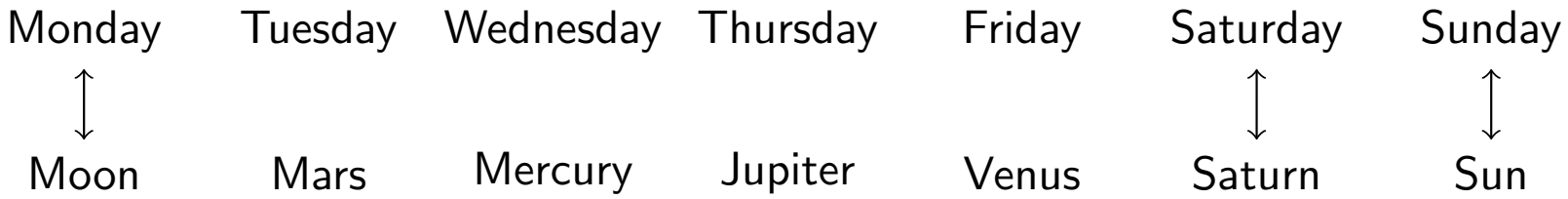

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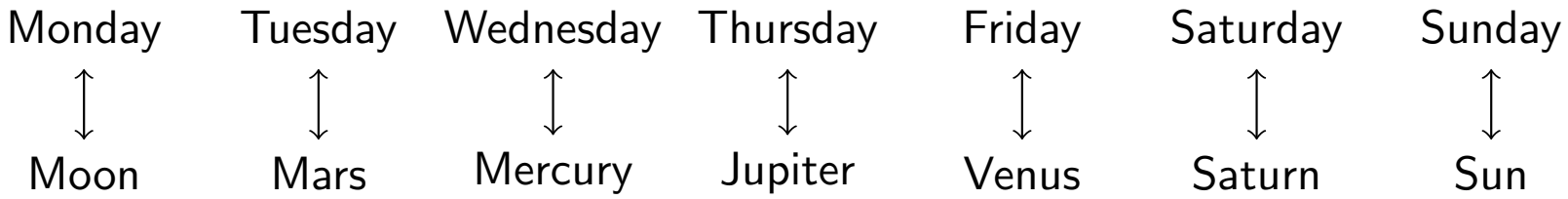

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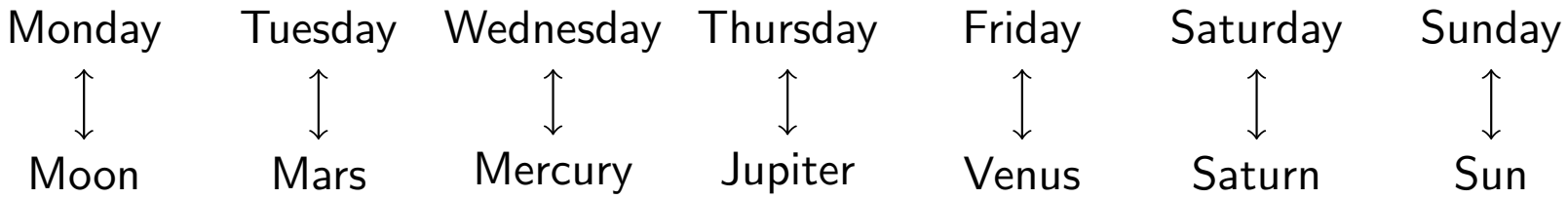

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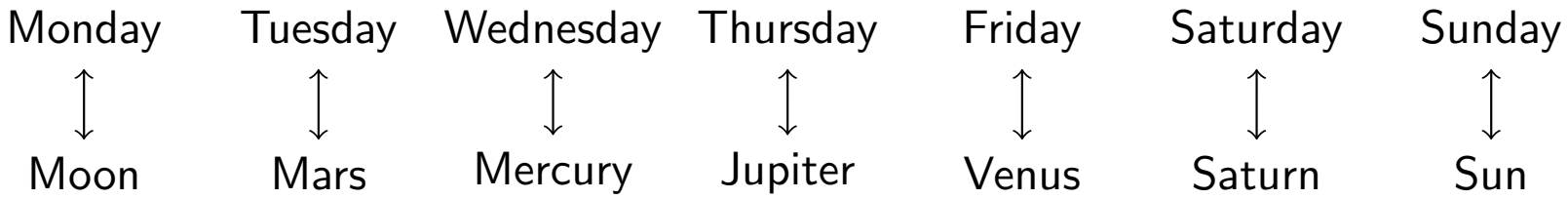

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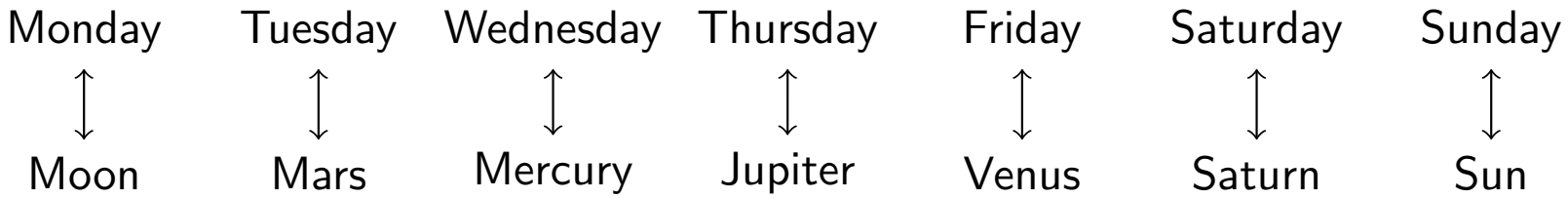

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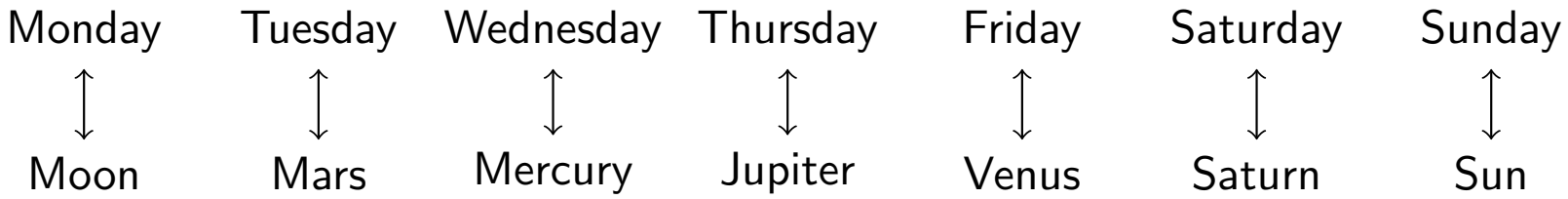

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  - Should differentiate between an object or concept and its name

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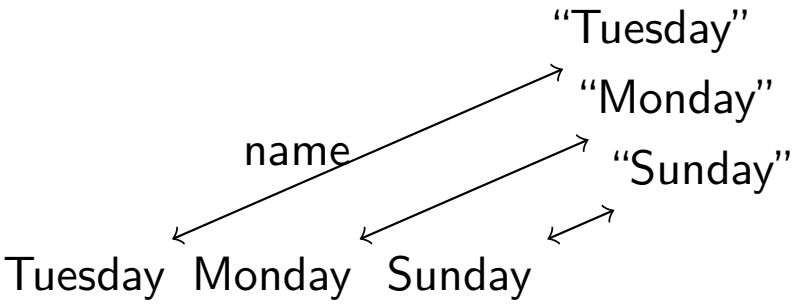

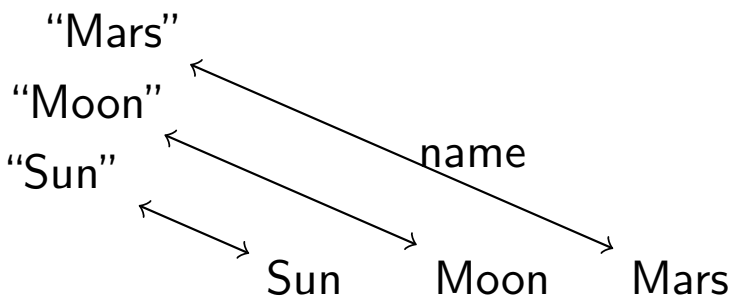

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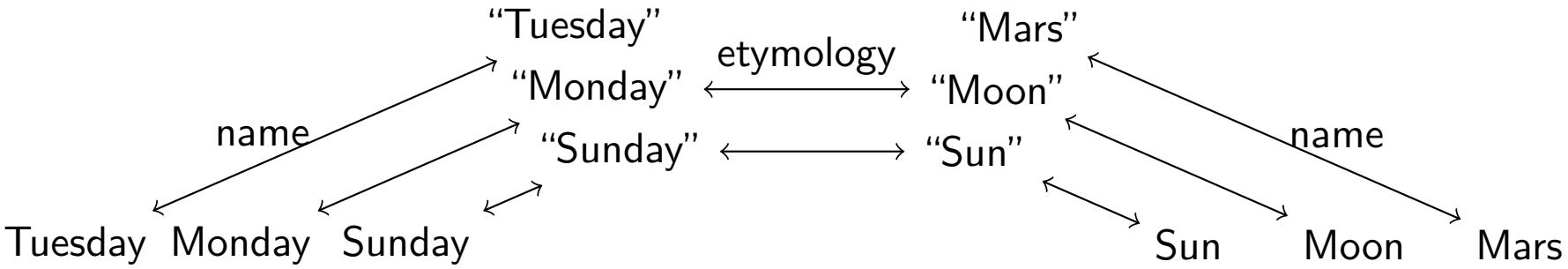

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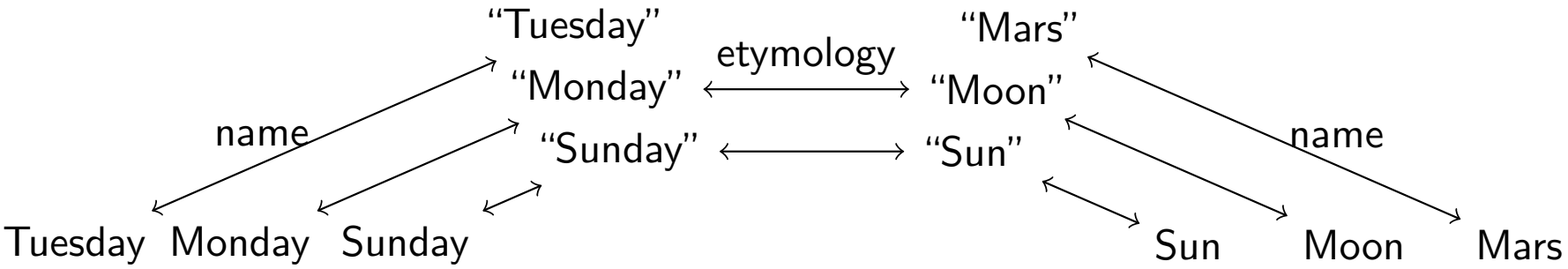

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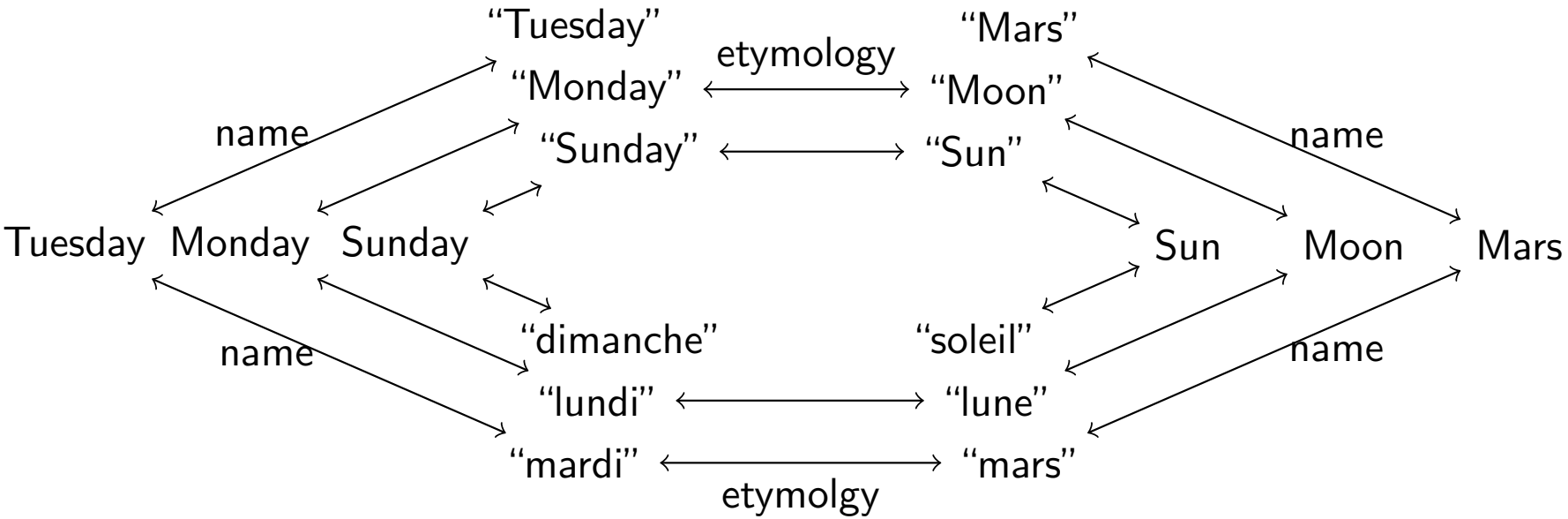

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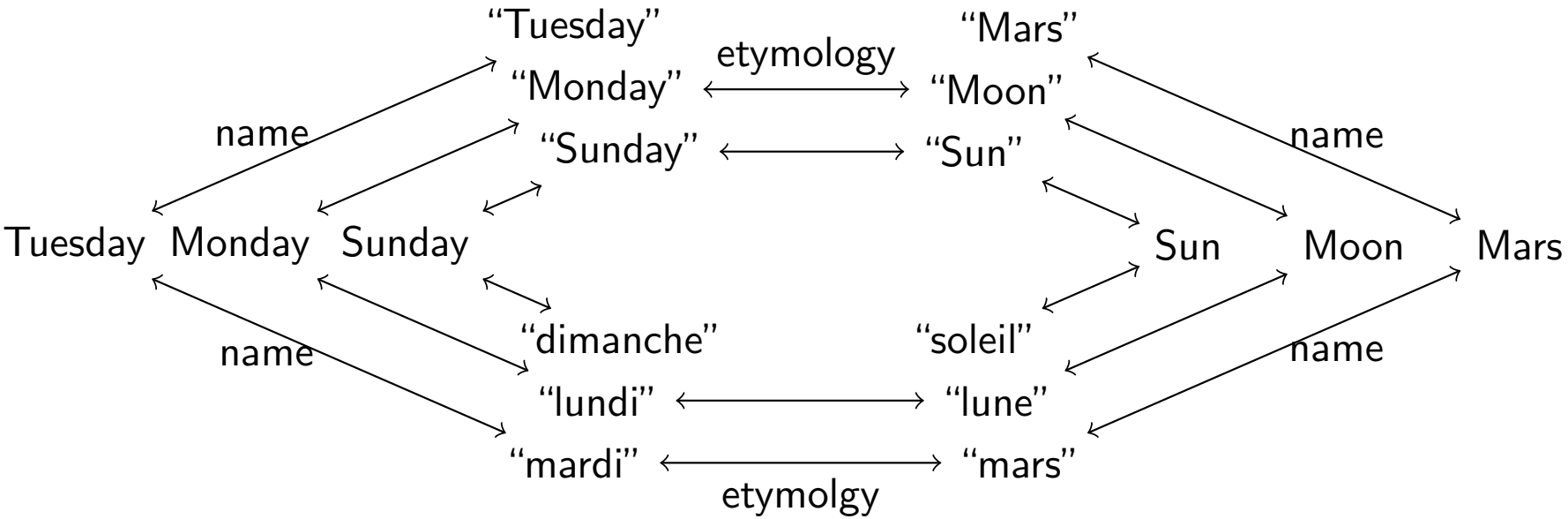

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- Consistent: Monday $\longleftrightarrow$ Moon through either language

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- Use commutative diagrams

Sets and functions between sets

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Some sets:

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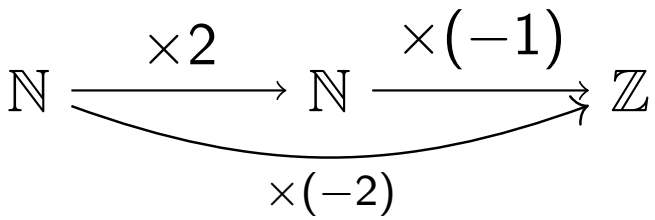

Can compose functions.

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$$D_{en} = \{\text{“Monday”}, \text{“Tuesday”}, \ldots\}, L_{en} = \{\text{“Moon”}, \text{“Mars”}, \ldots\}$$
$$D_{fr} = \{\text{“lundi”}, \text{“mardi”}, \ldots\}, L_{fr} = \{\text{“lune”}, \text{“Mars”}, \ldots\}$$

etc.

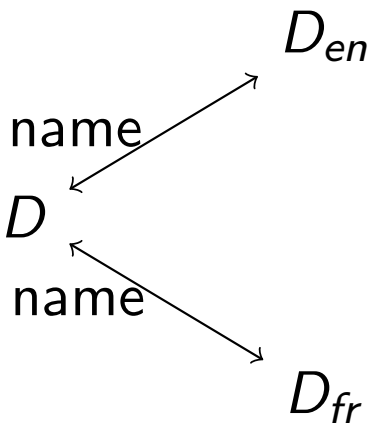
- $X \longleftrightarrow Y$ means a correspondence, i.e. a bijection $X \to Y$
- $X \dashleftarrow\dashrightarrow Y$ means a partial correspondence

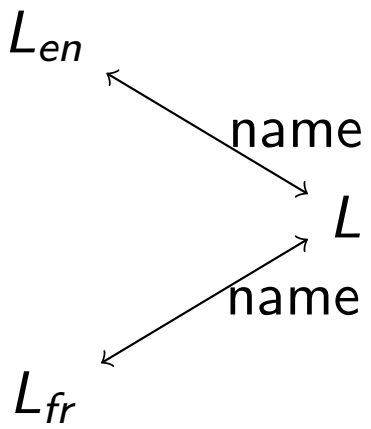
New diagram

New diagram

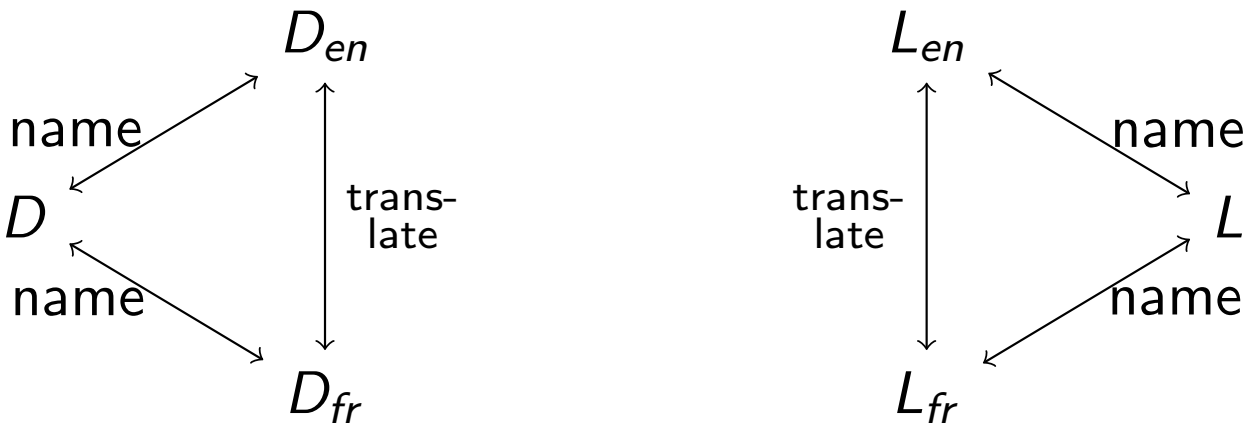
D L

New diagram





New diagram



- The two triangles commute (definition of translation)

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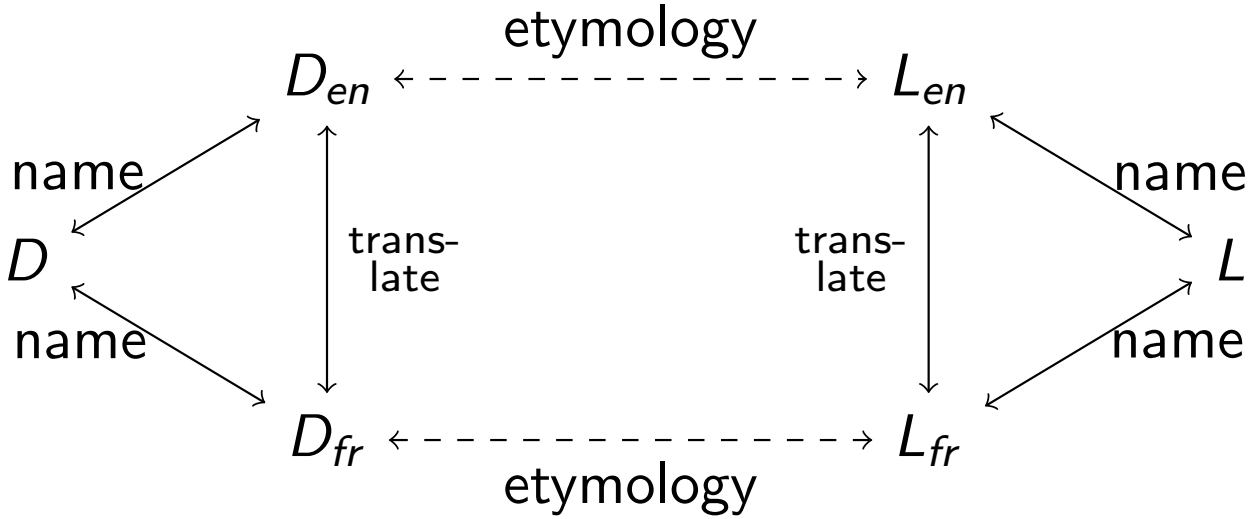

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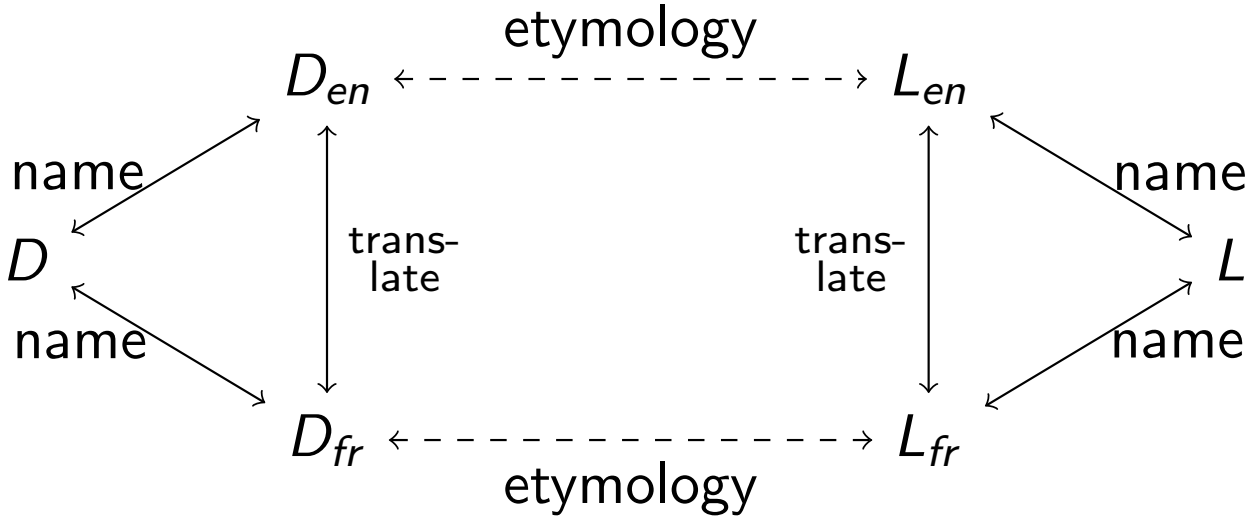

- The two triangles commute (definition of translation)
- The rectangle commutes (i.e. consistent on Monday $\longleftrightarrow$ moon)
- Planetary days $D \longleftrightarrow L$ obtained by combining the two partial correspondences

A single language

Latin

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Latin

dies Lūnae dies Martis dies Mercurīī dies Iovis dies Veneris dies Saturnī dies Sōlis

A single language

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- Can realize the correspondence $D \longleftrightarrow L$ through Latin alone

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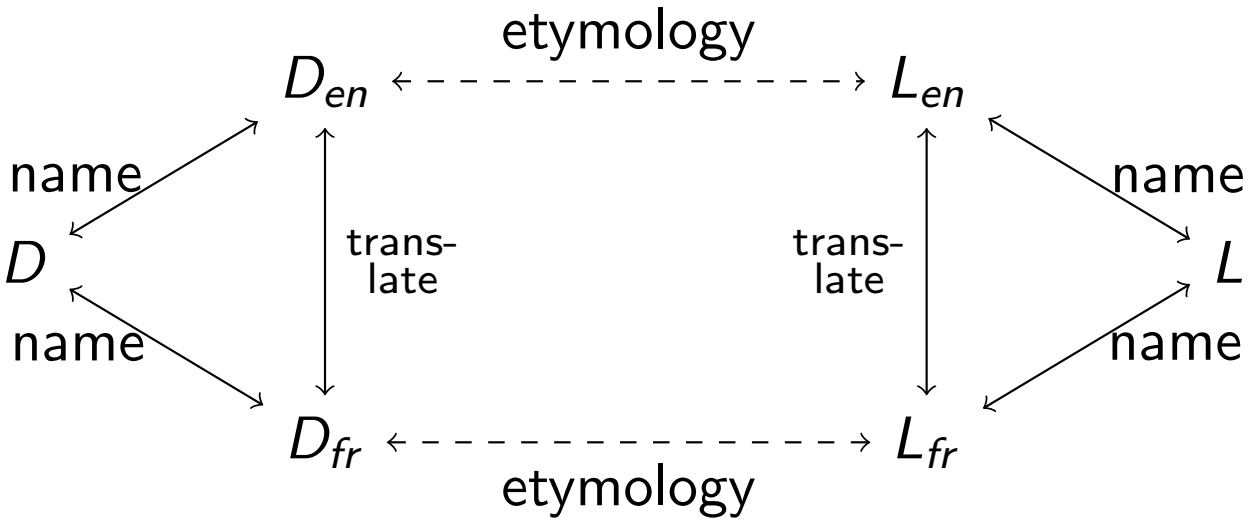

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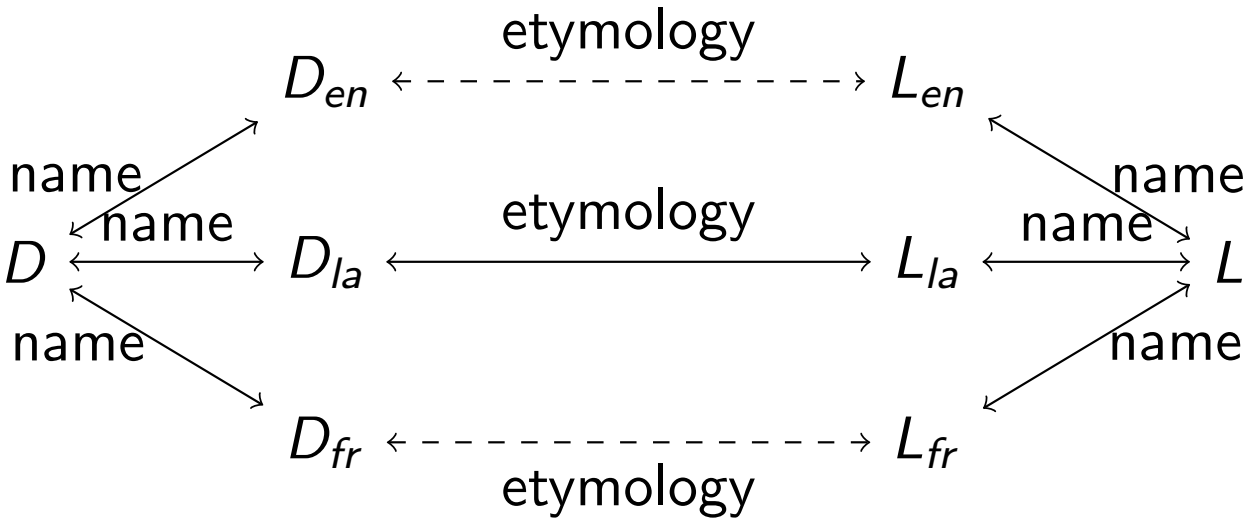

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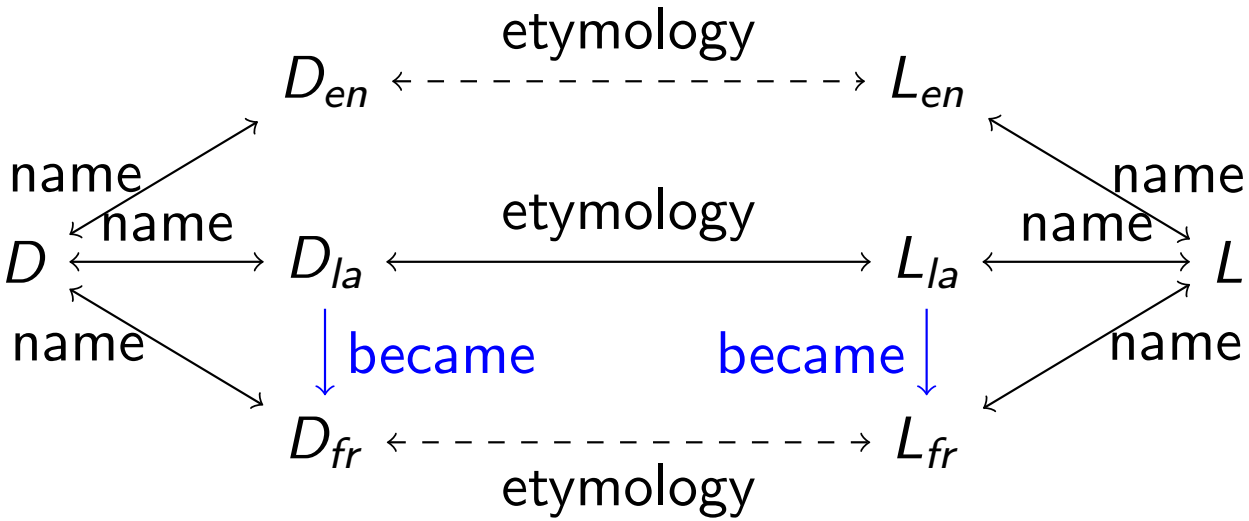

- Translation refined to descendant relation la $\longrightarrow$ fr

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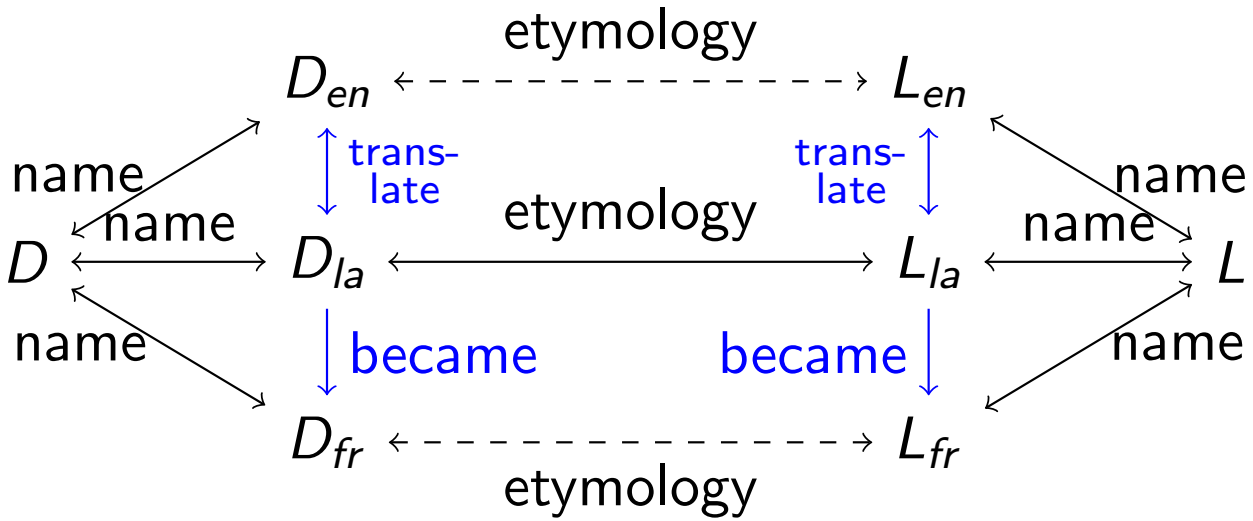

- Translation refined to descendant relation la $\longrightarrow$ fr
- la $\longleftrightarrow$ en is still mysterious

Enter the gods

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- D and L were named after Roman gods

$$G_R = \{\text{Roman gods used to name } D \text{ and } L\}$$

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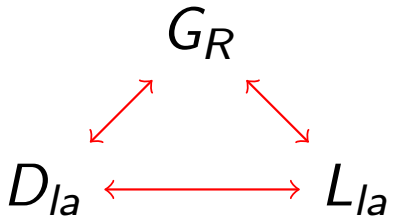
- Triangle



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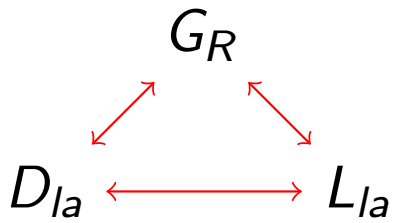
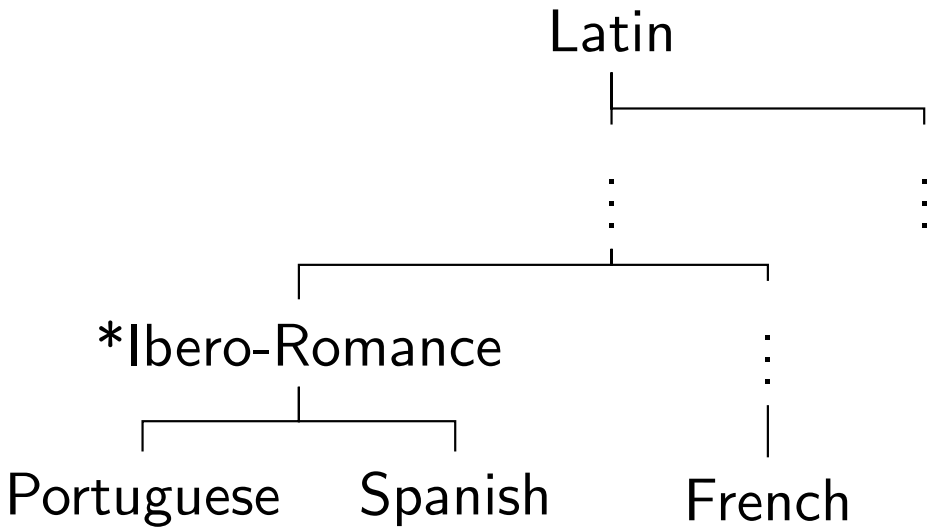


- This can be traced through languages

The triangle for Romance languages

The triangle for Romance languages

A very simplified family tree of Romance languages:



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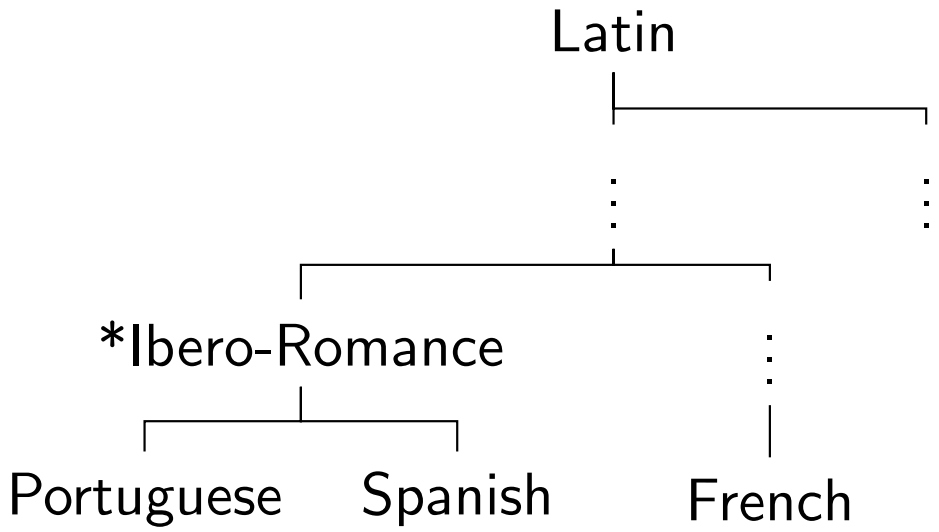

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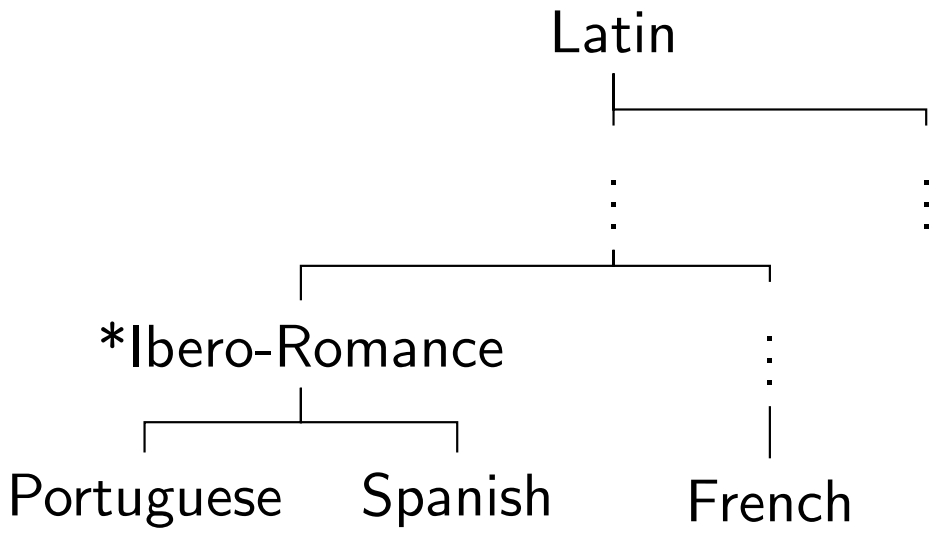
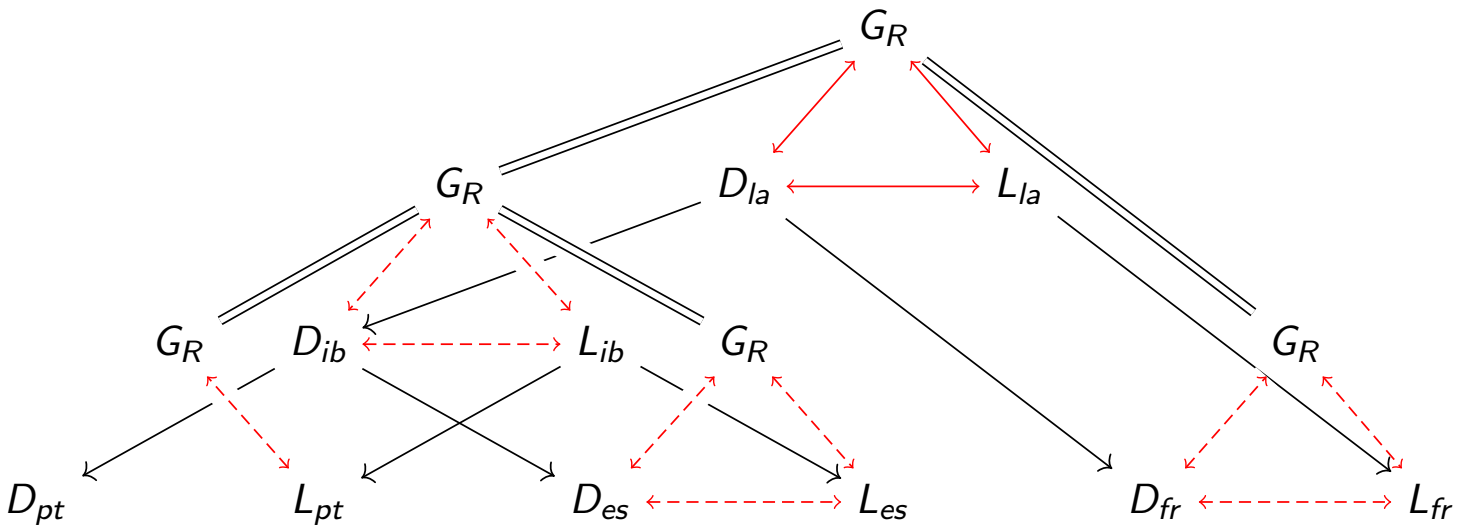


- All descended from Latin
- (*Ibero-Romance may not actually have existed)

The triangle for Romance languages

Triangle at each language in the tree:



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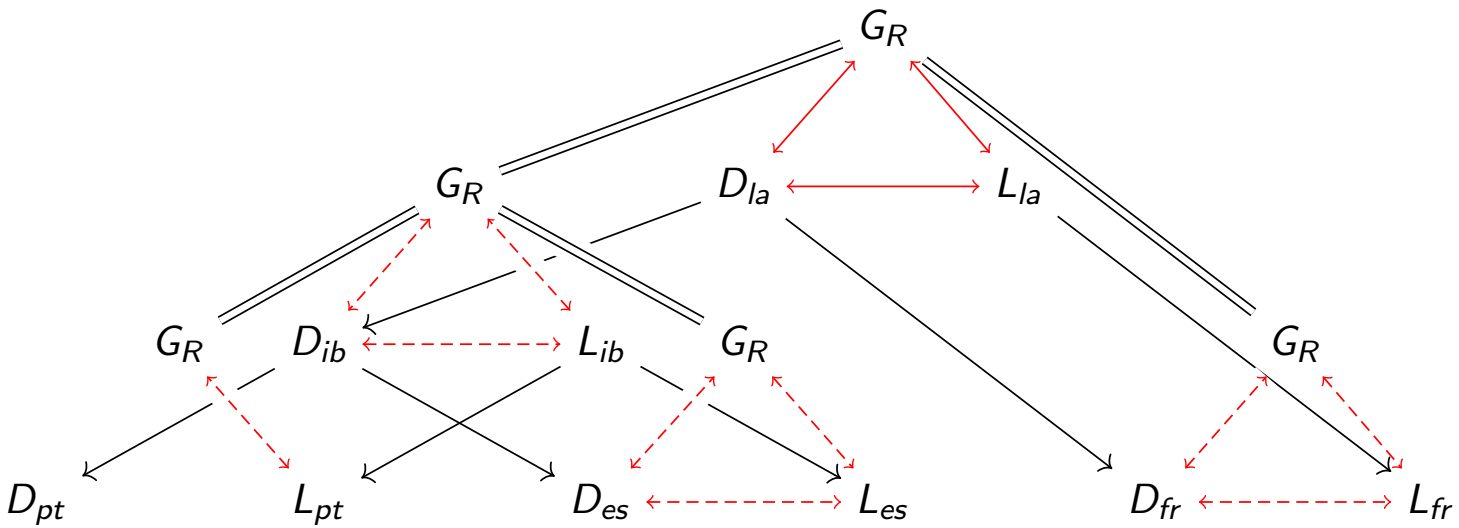

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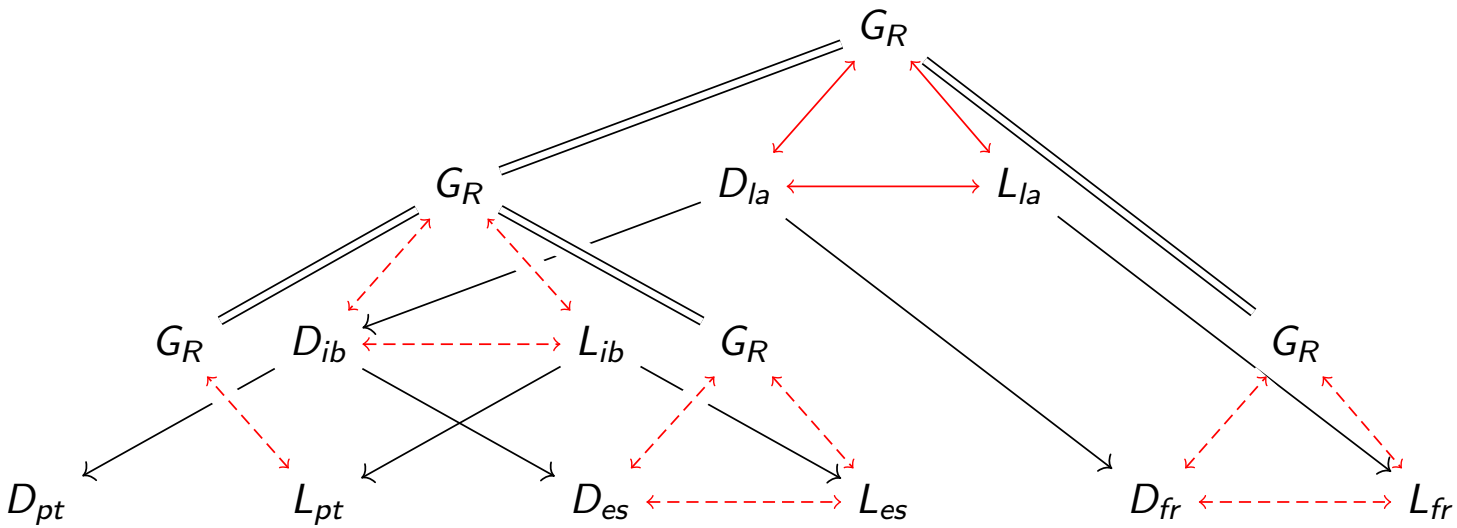

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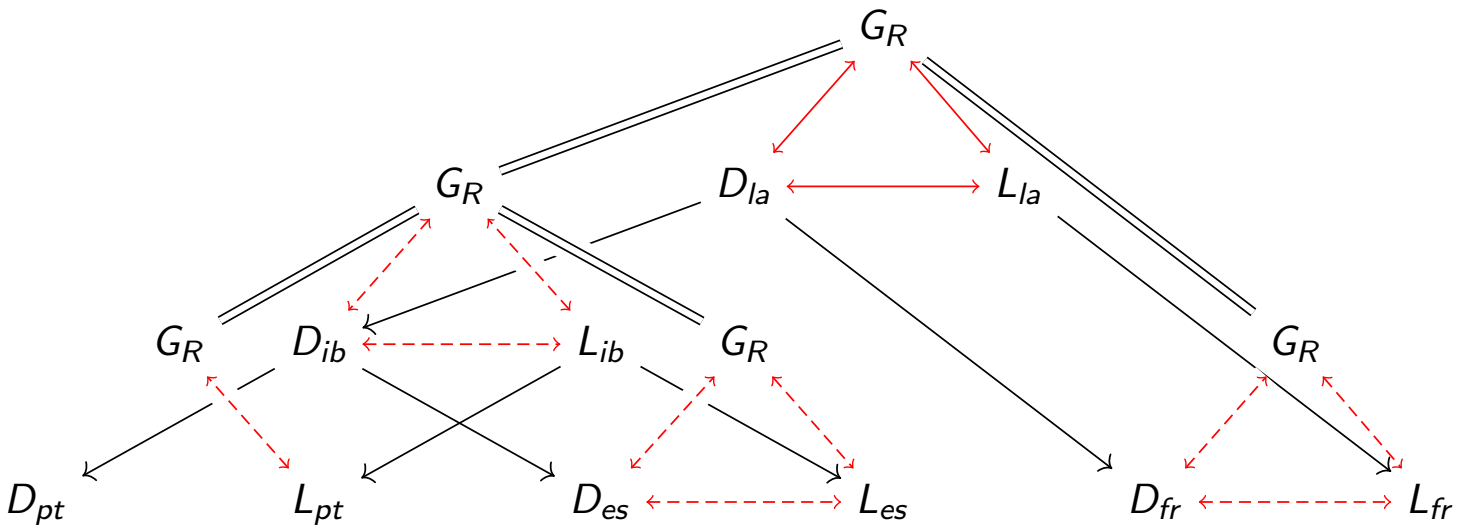

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- Lost for Saturday and Sunday in French, Spanish
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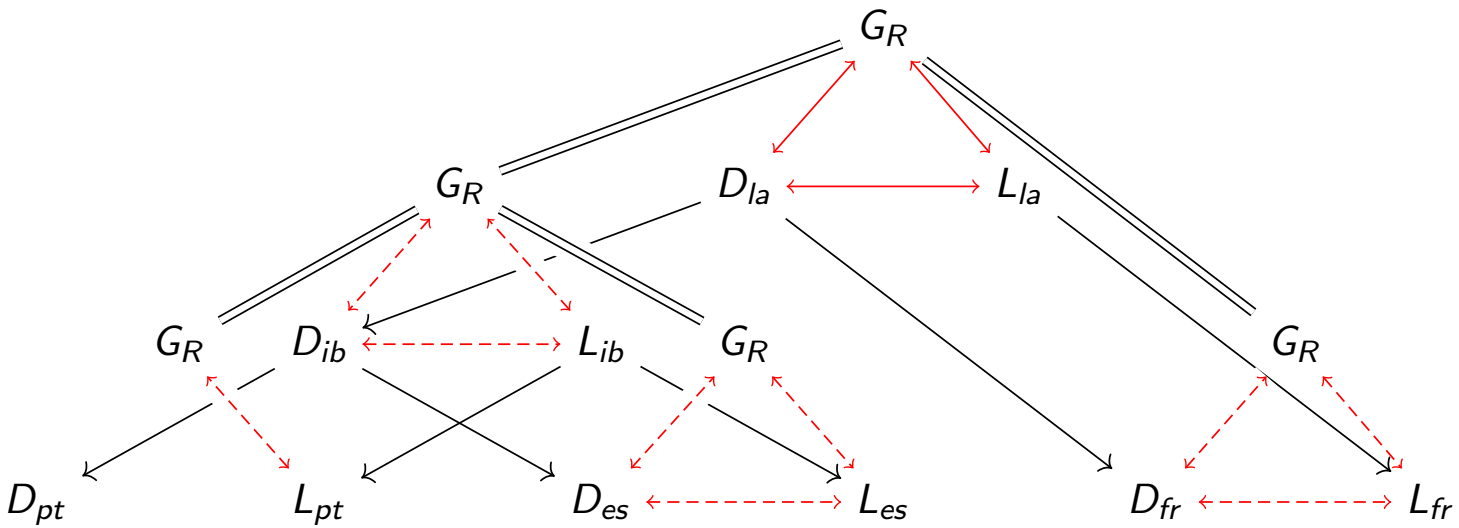

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- Partial correspondences for *Ibero-Romance must have been at least as strong as for Spanish

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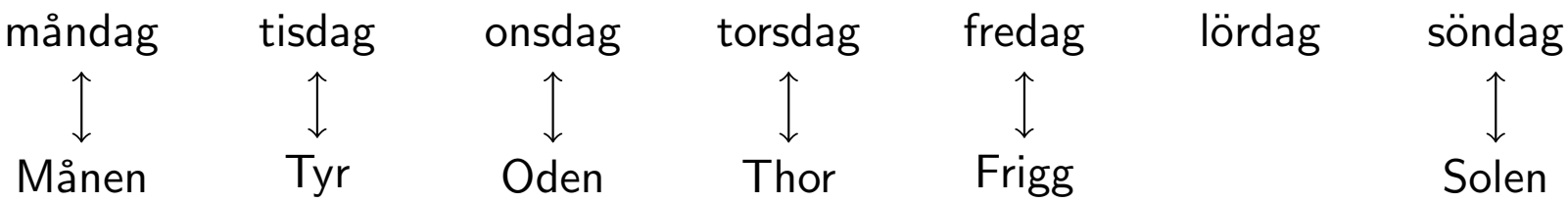

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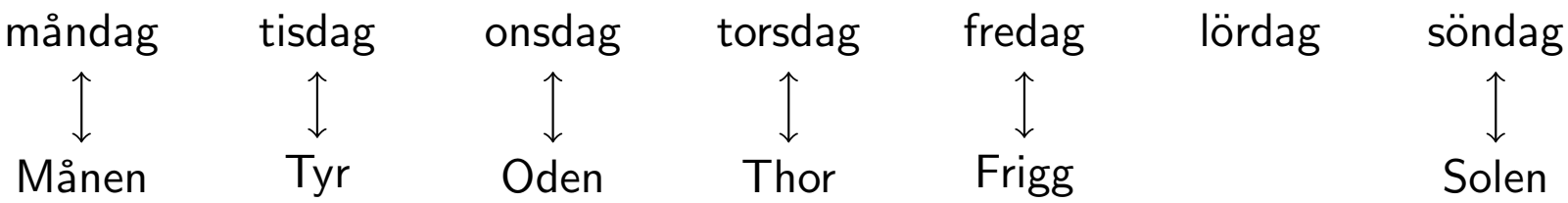

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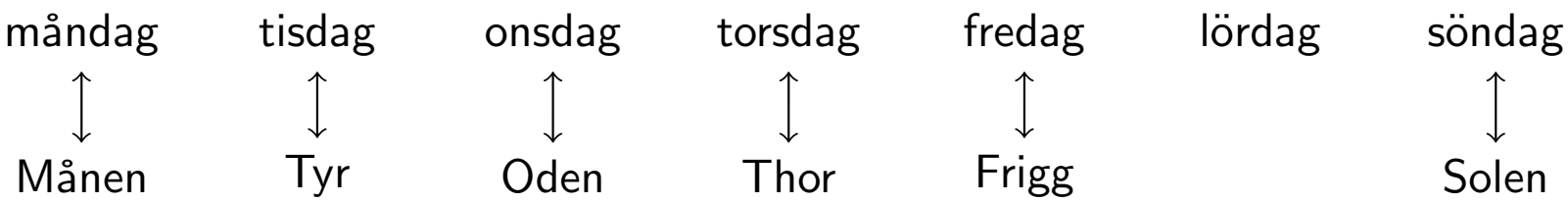

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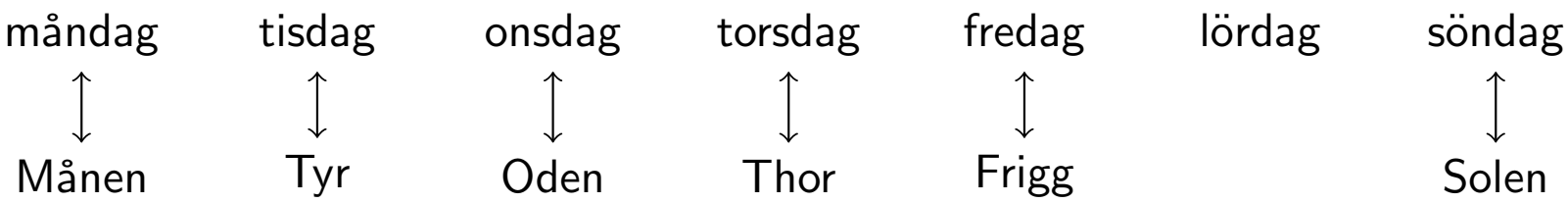

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  - *Proto-Germanic *Wōdanaz, Proto-Norse Wōdin, Old Norse Ōdhinn
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- Without history, can't say where the triangle originated, but it must already have existed in proto-Germanic

Some history

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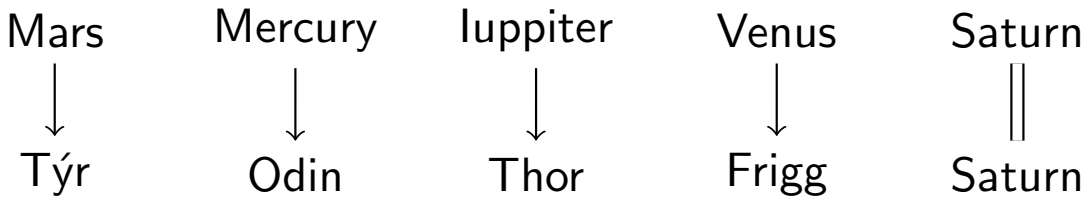

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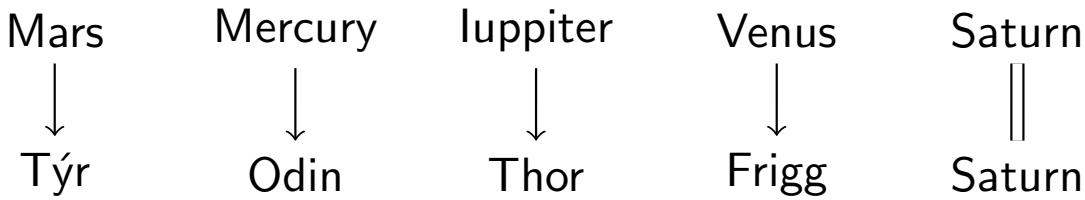

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- Choice: How did they choose the corresponding Germanic god?
- **Interpretatio germanica**: "[T]he practice by the Germanic peoples of identifying Roman gods with the names of Germanic deities." (Wikipedia)

A surprising language: Japanese

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月曜日　火曜日　水曜日　木曜日　金曜日　土曜日　日曜日

- 曜日(Luminary Day)

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月曜日　火曜日　水曜日　木曜日　金曜日　土曜日　日曜日

月　火星　水星　木星　金星　土星　太陽

- 曜日(Luminary Day)，星(Star)

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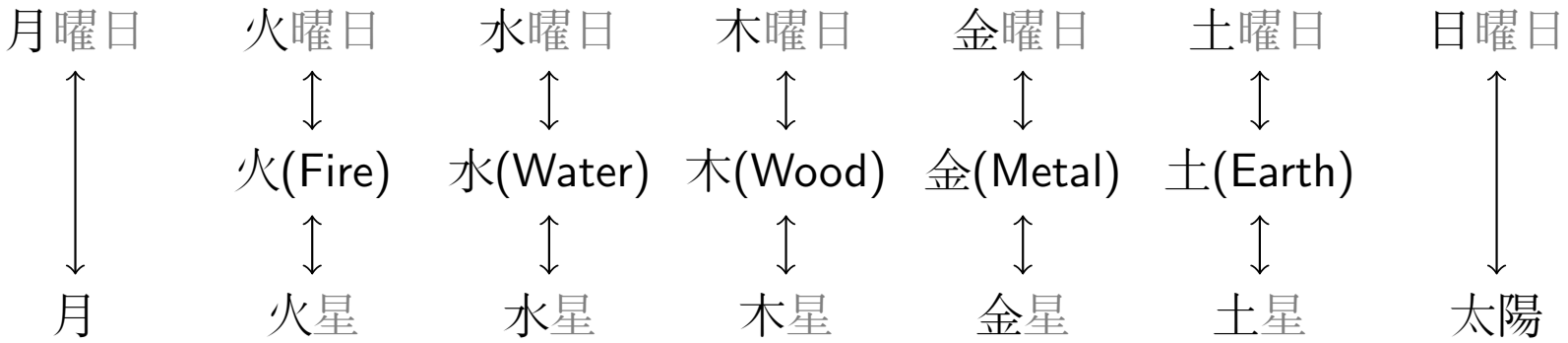

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- Other days and luminaries: $E = \{\text{Five Elements of Chinese thought}\}$

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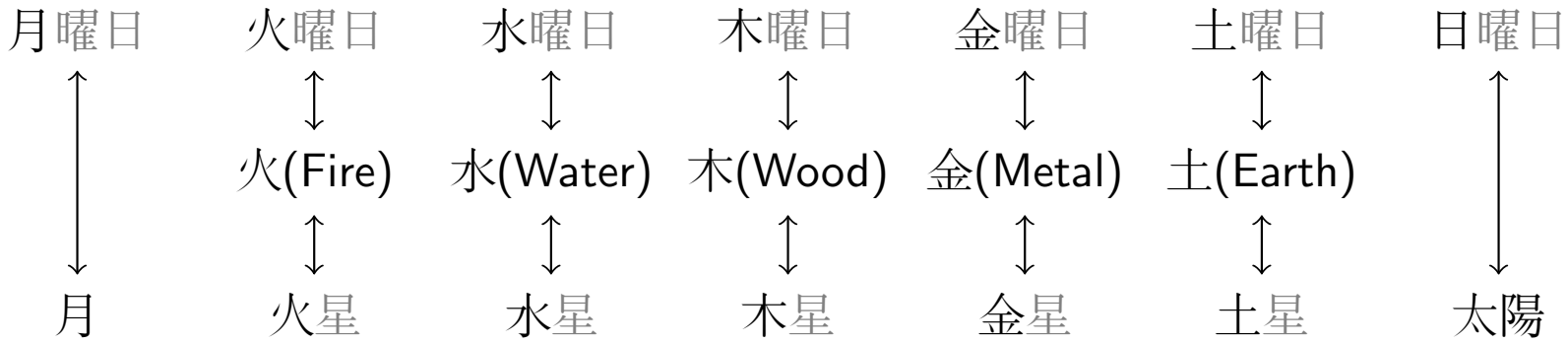

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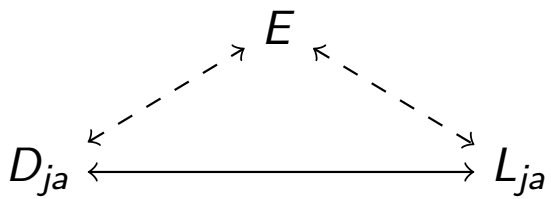

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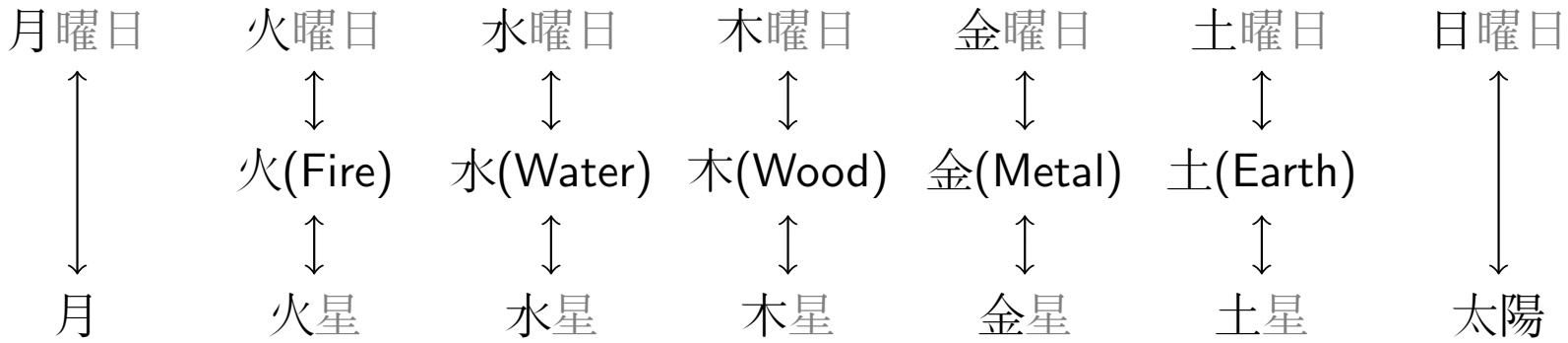

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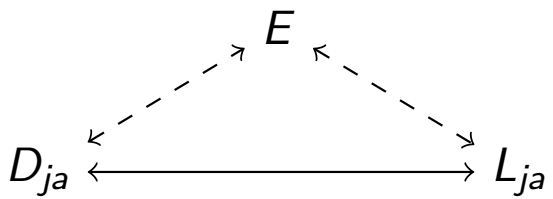

- This is the same correspondence $D \longleftrightarrow L$!

A surprising commutative diagram

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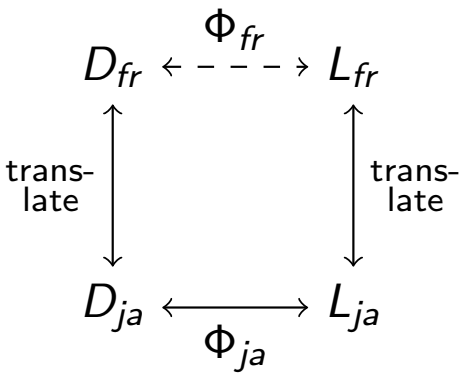

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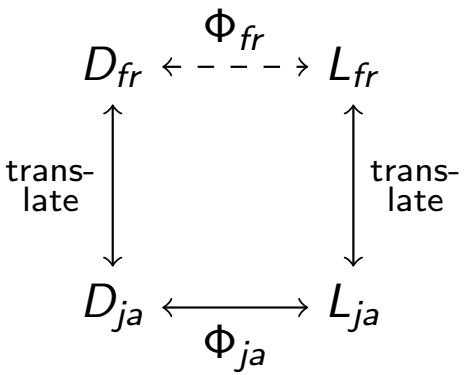

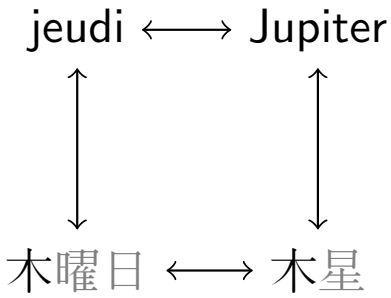

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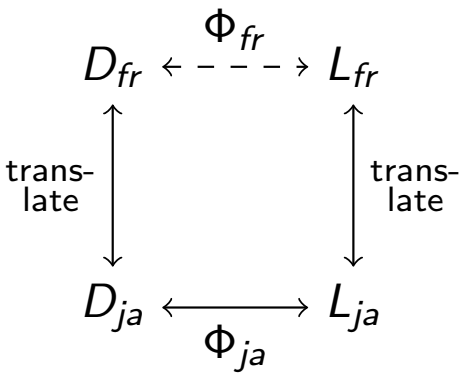

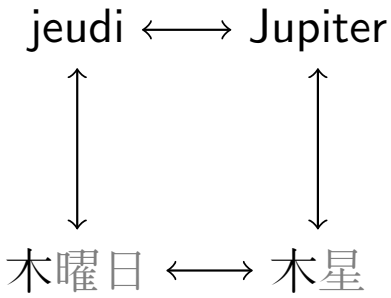

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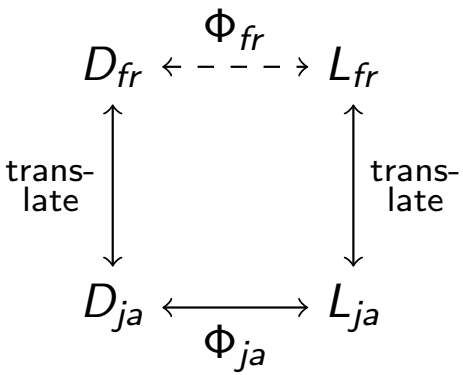

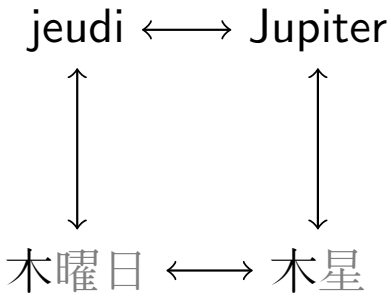

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- E has an order (unlike G_R), but unrelated to the order on D

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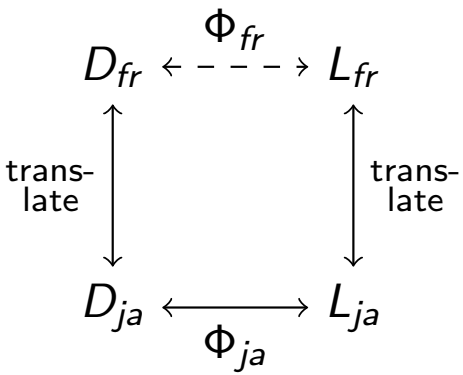

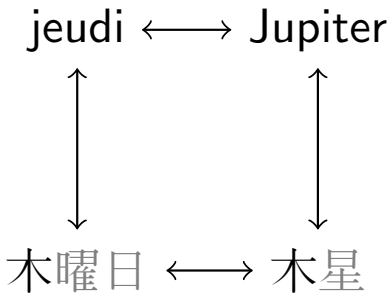

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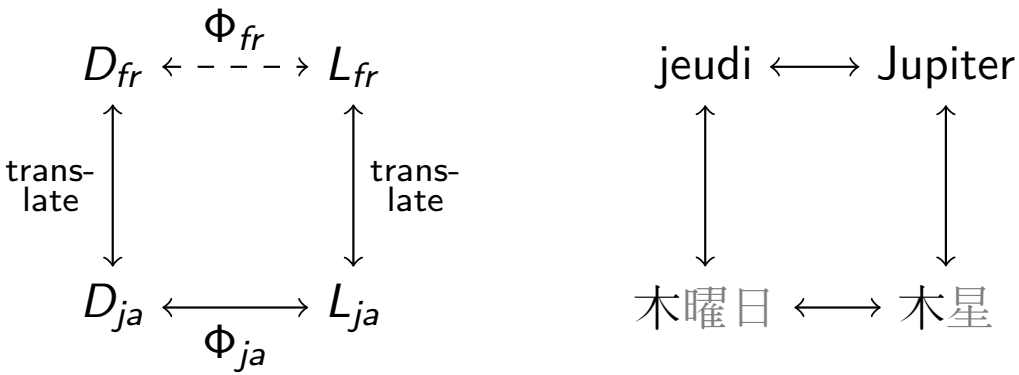
A surprising commutative diagram



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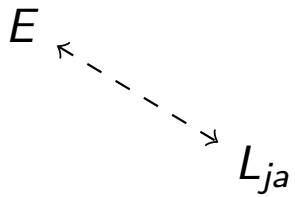

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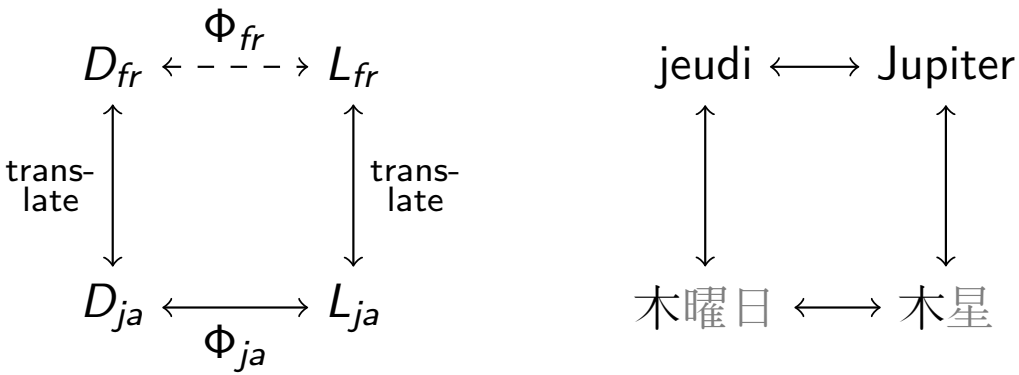

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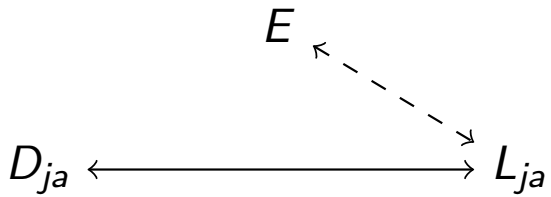

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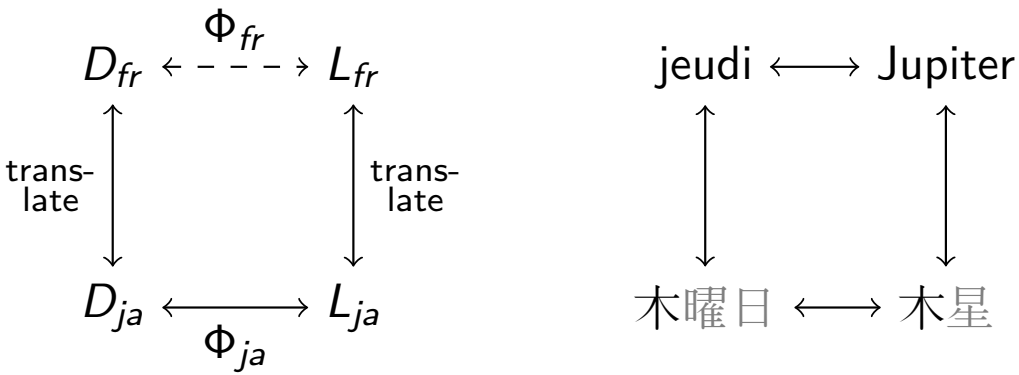

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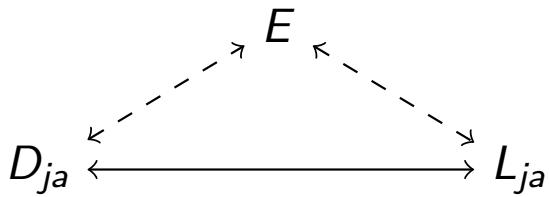

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Application: Me

- What correspondences exist in my head?

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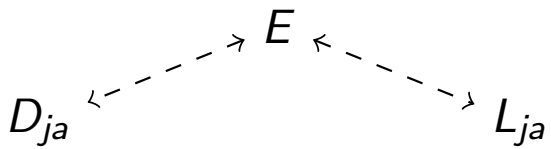

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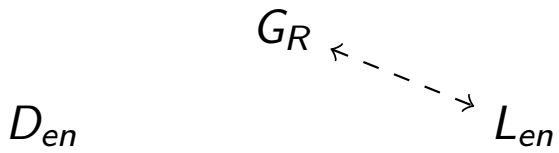

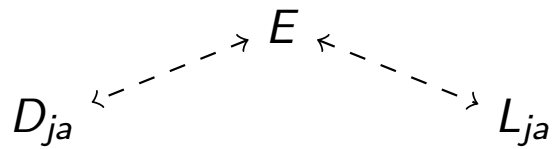

- Native Japanese speaker who knows English

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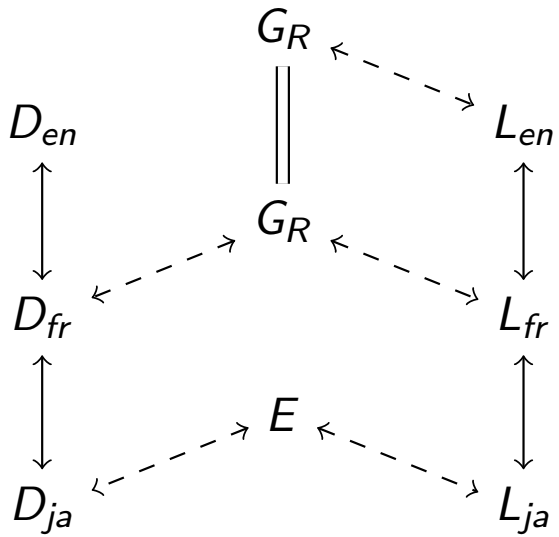

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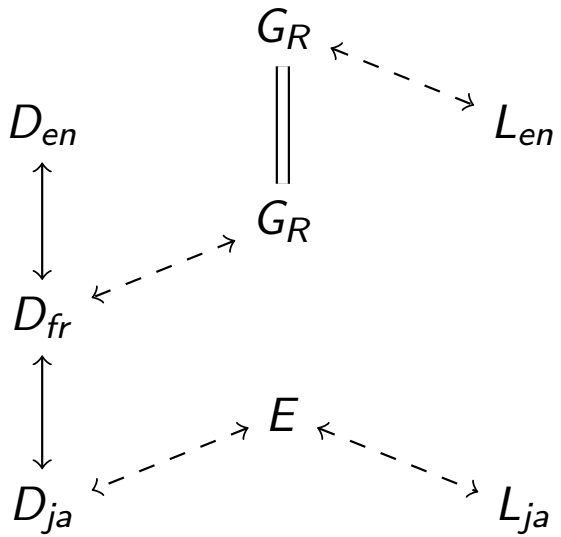

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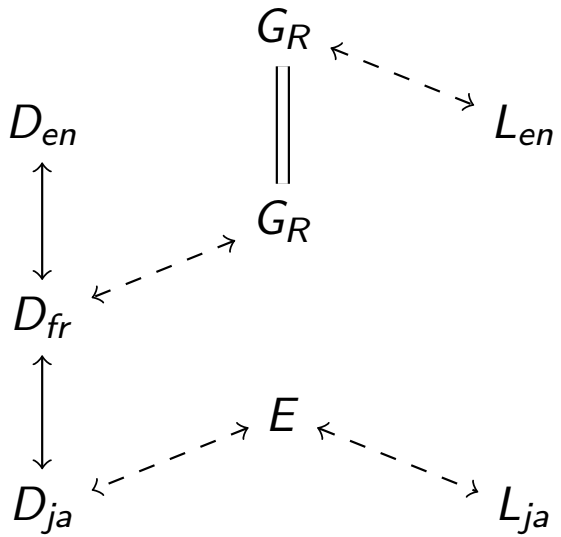

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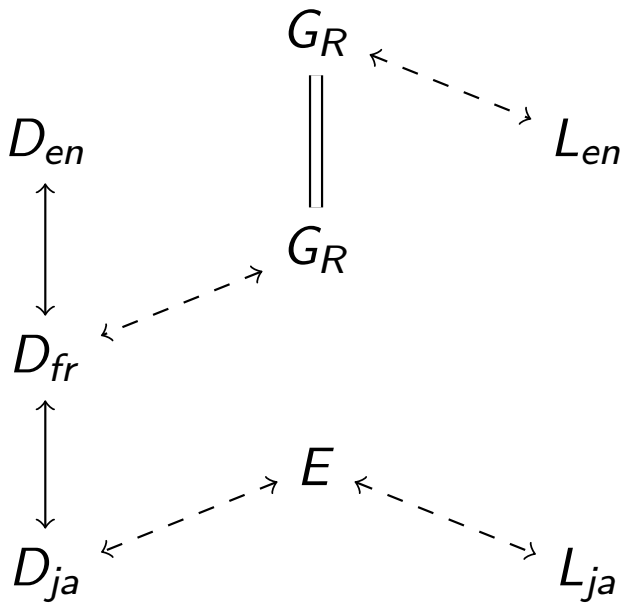

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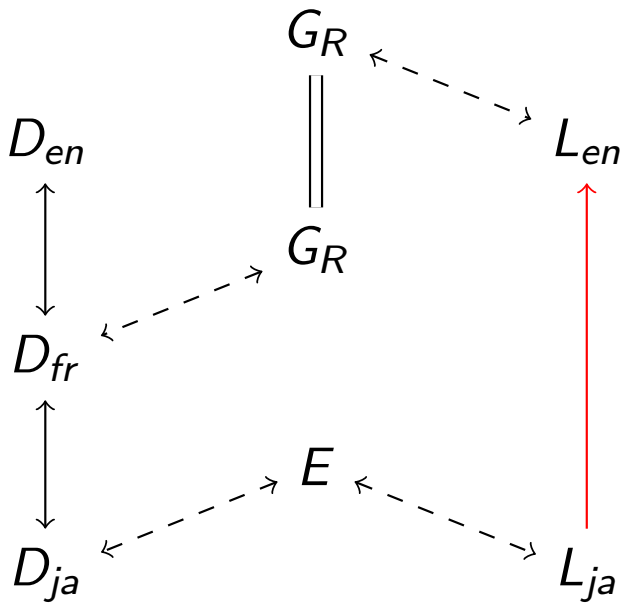

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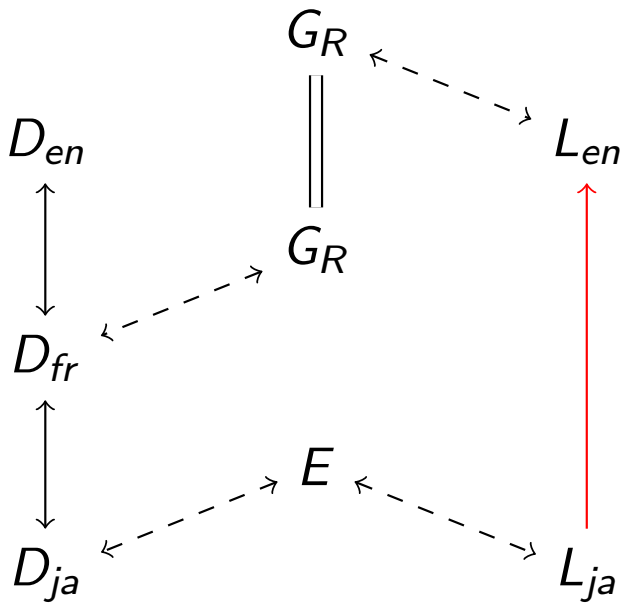

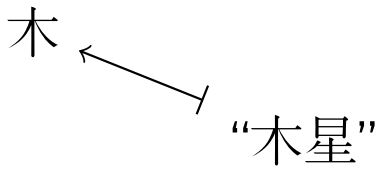

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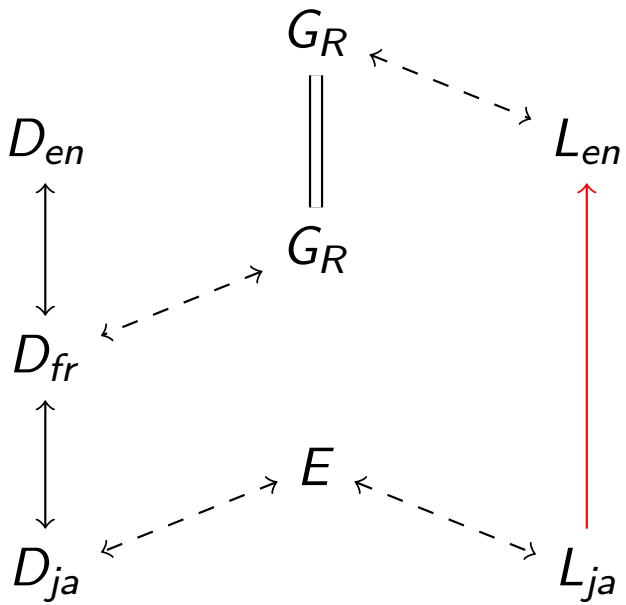

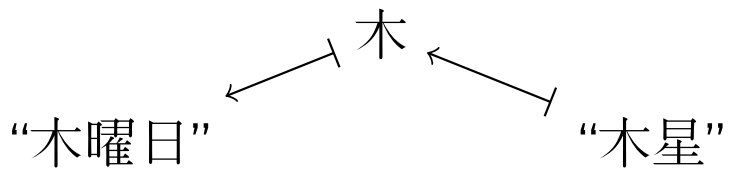

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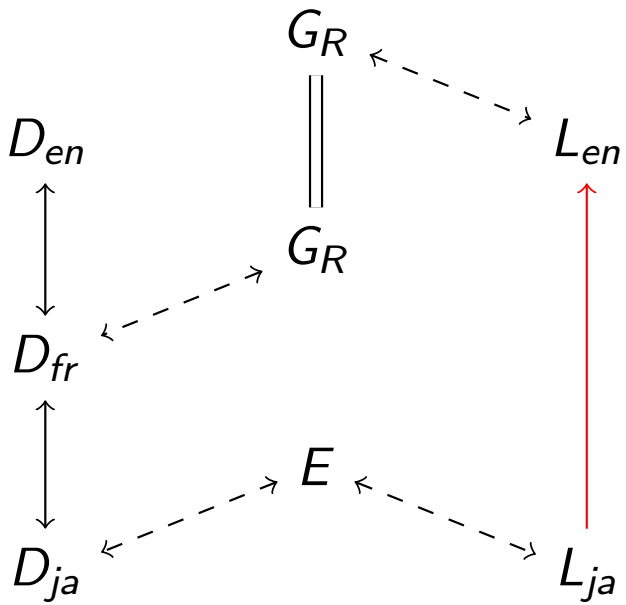

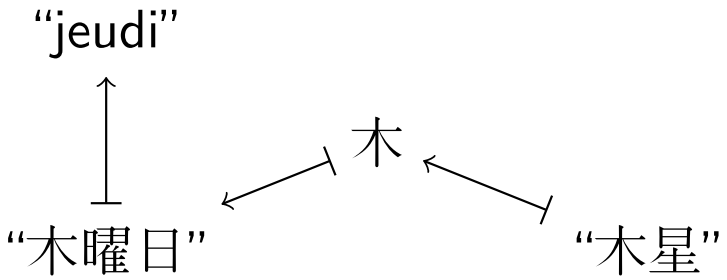

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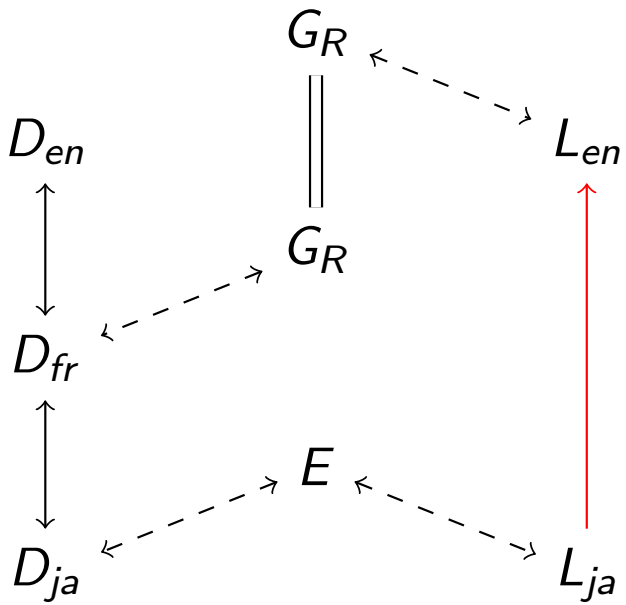

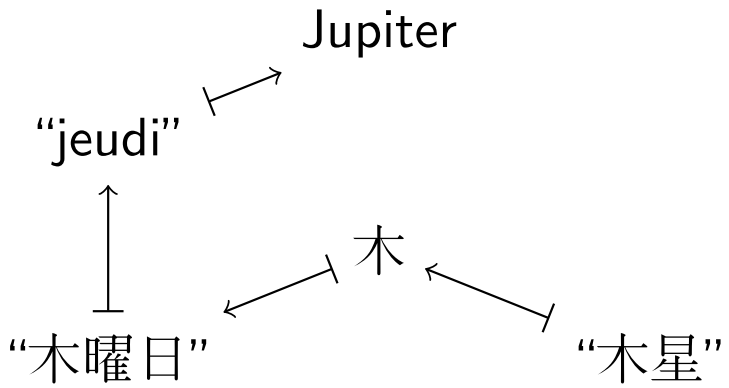

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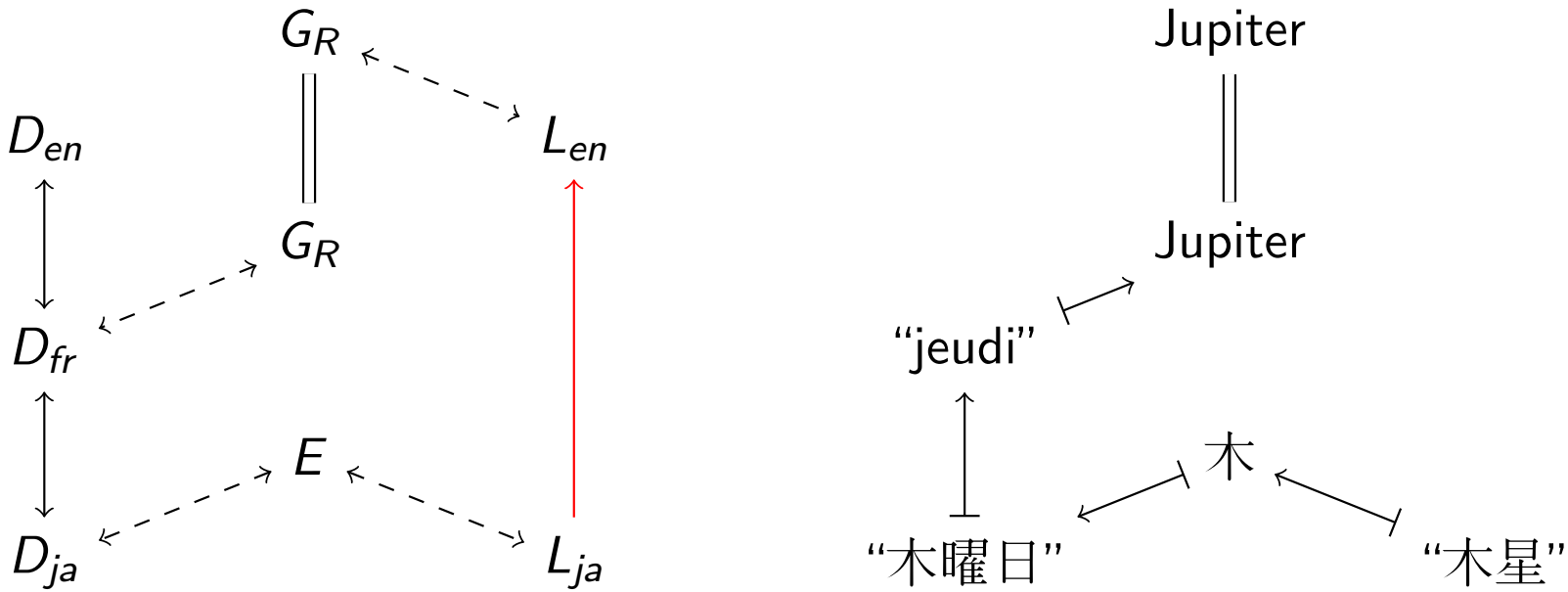

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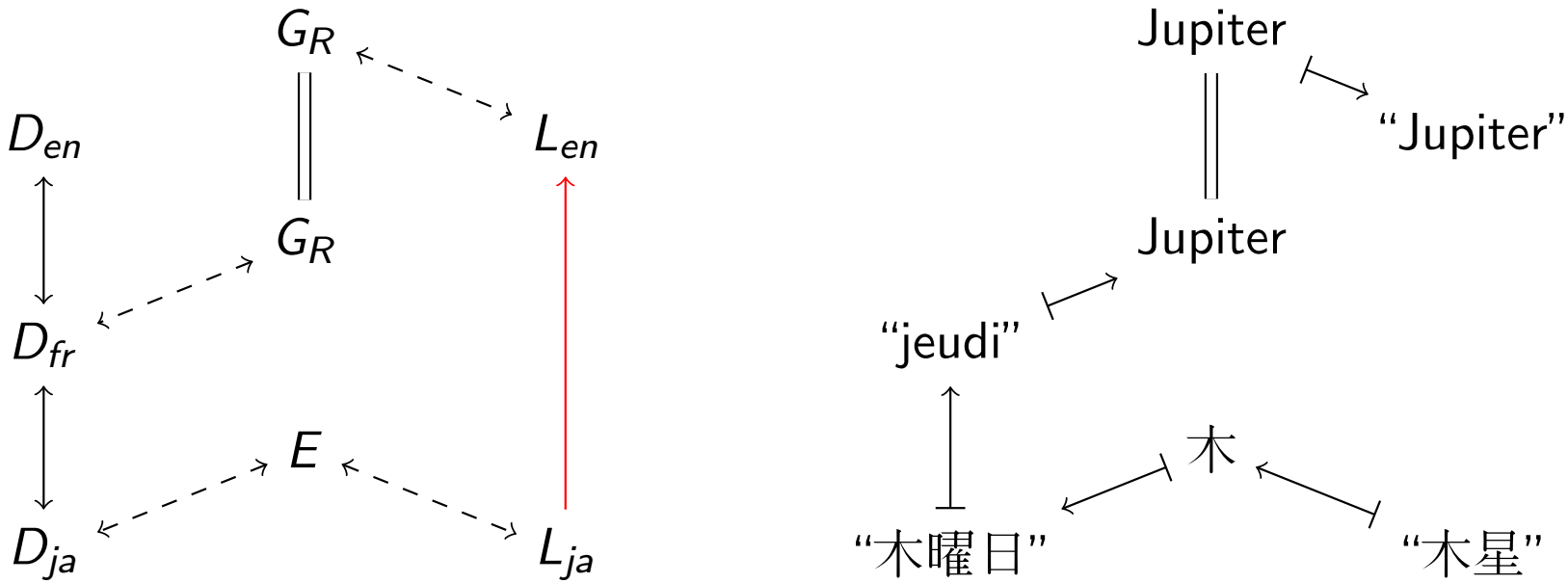

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Through China. But in Chinese:

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- 星期(Star period)

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星期一	星期二	星期三	星期四	星期五	星期六	星期日
月	火星	水星	木星	金星	土星	太阳

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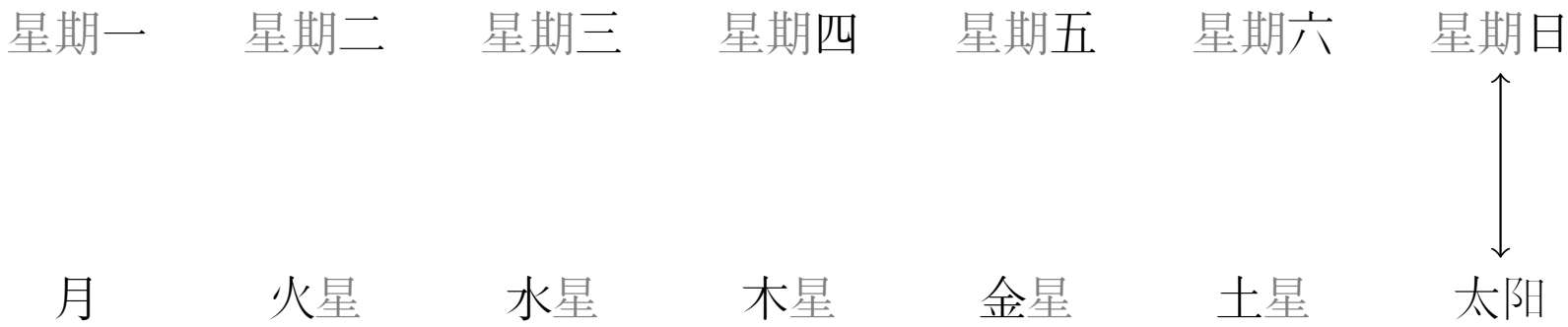

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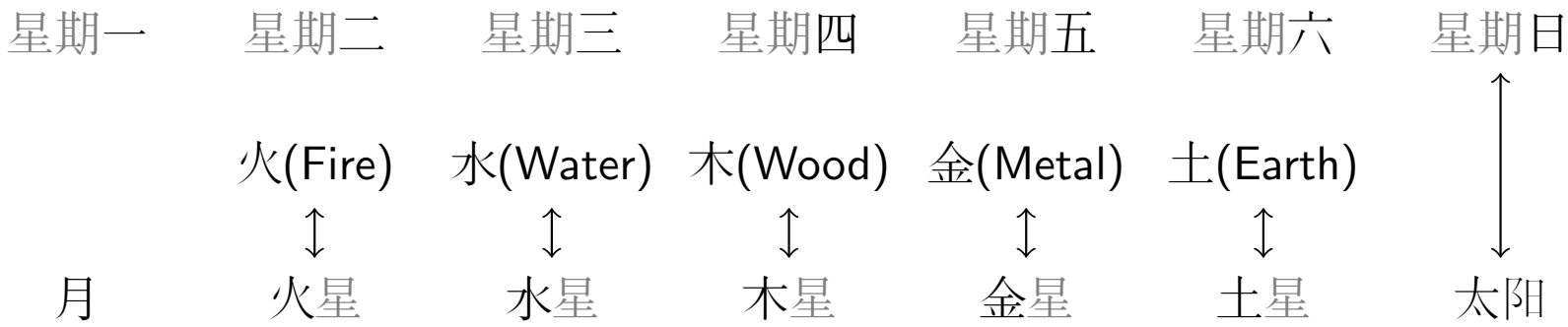

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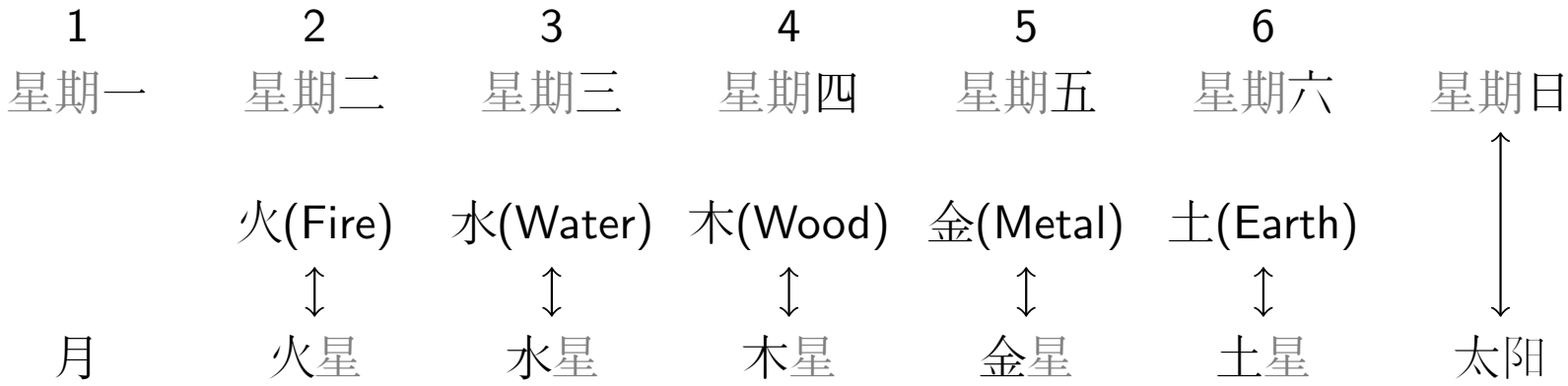

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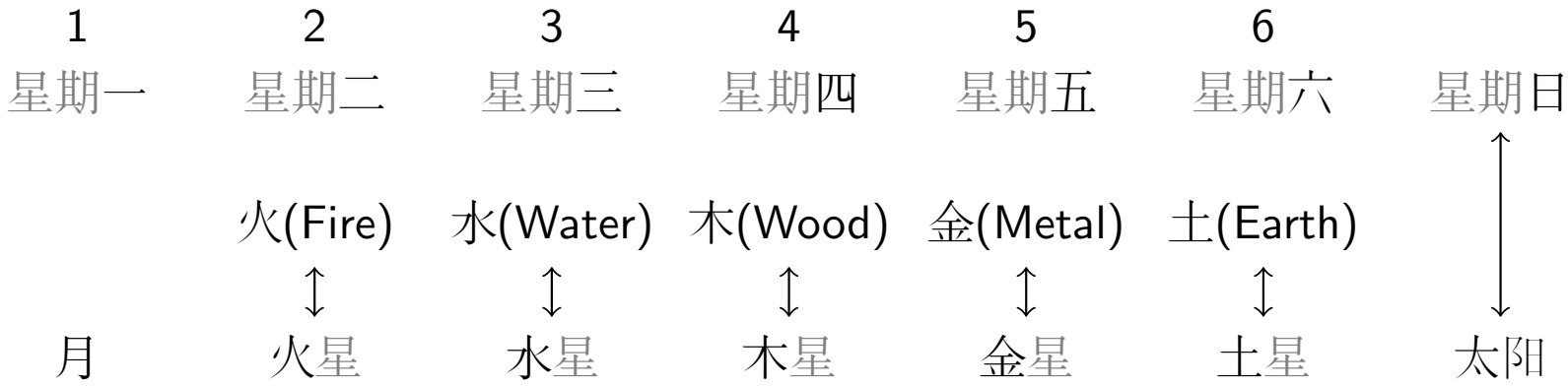

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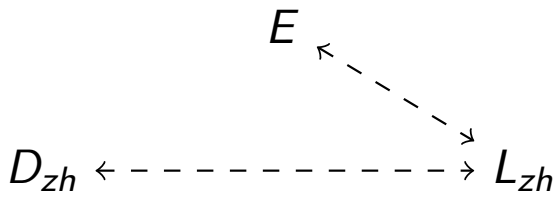

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Another application

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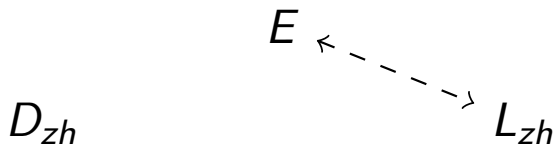
Someone who knows Chinese



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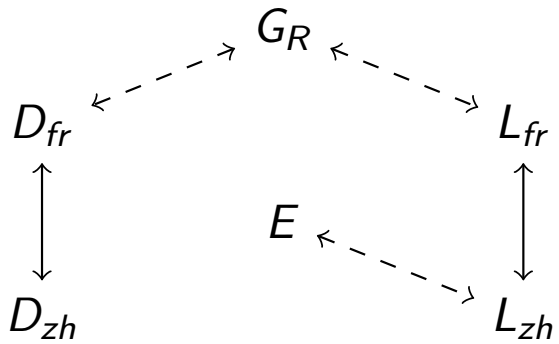
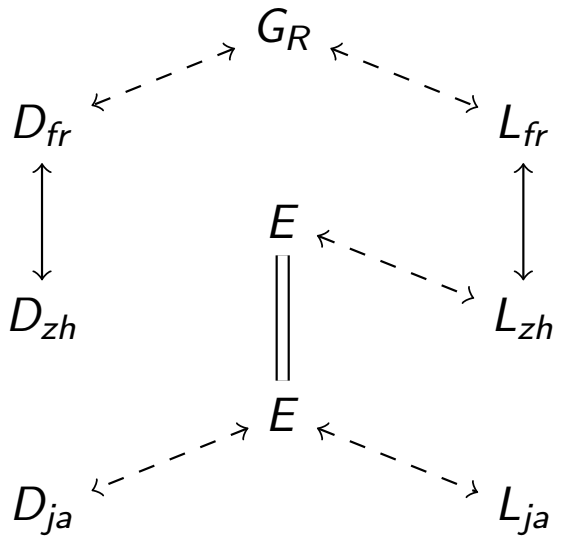
Someone who knows Chinese and (say) French



Another application

Someone who knows Chinese and (say) French can figure out the days of the week in Japanese.



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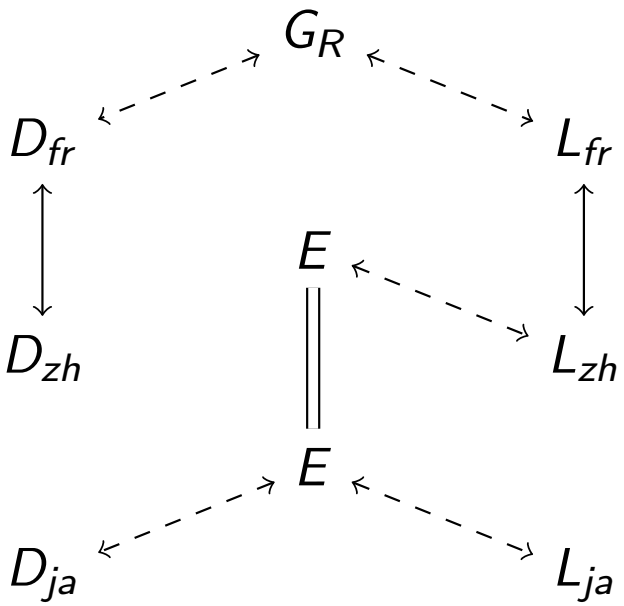

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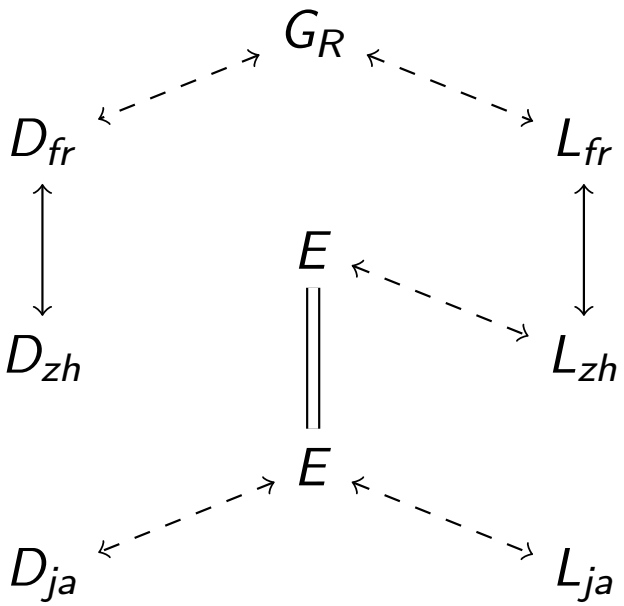

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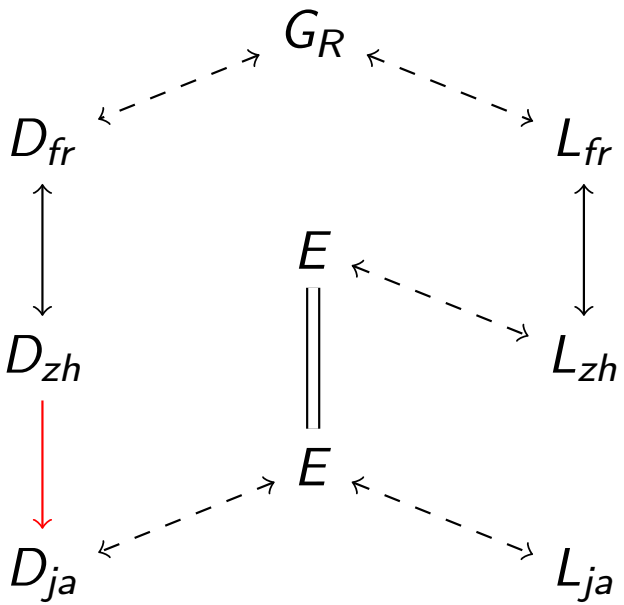

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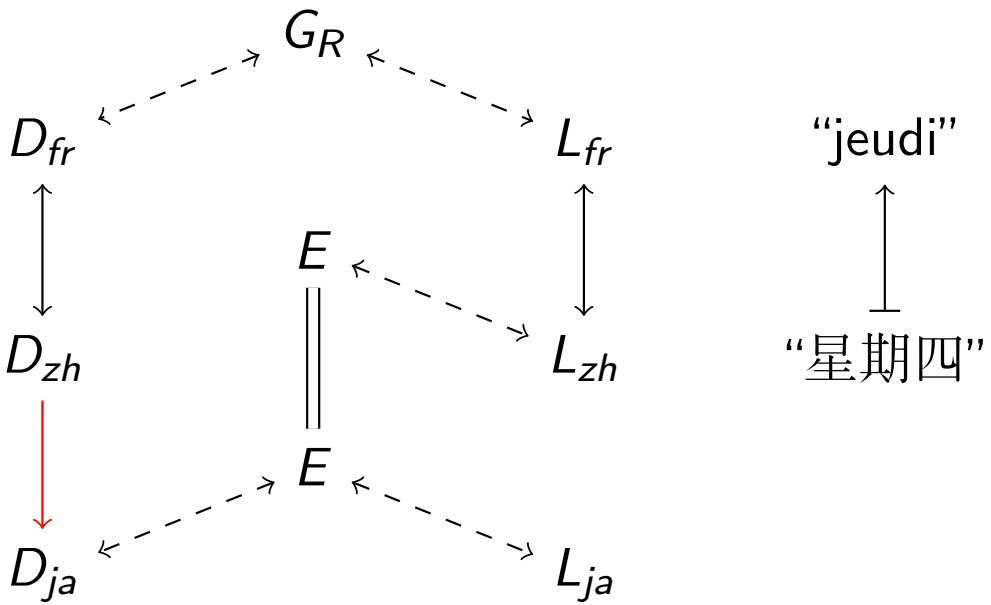

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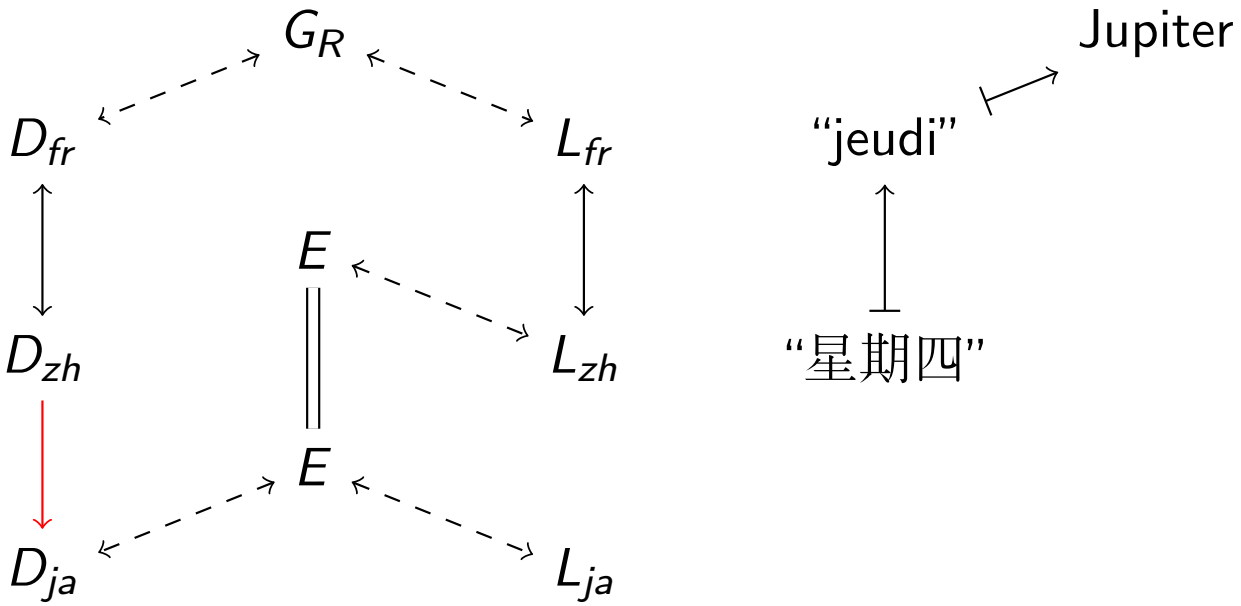

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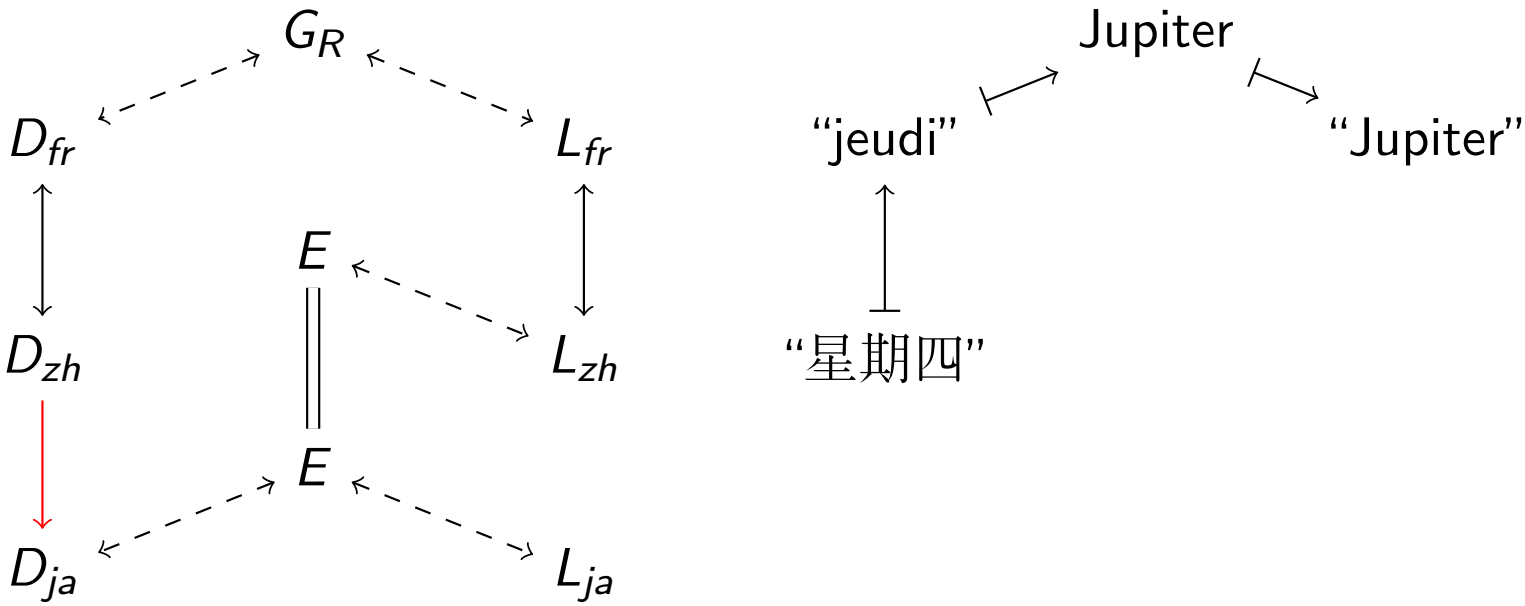

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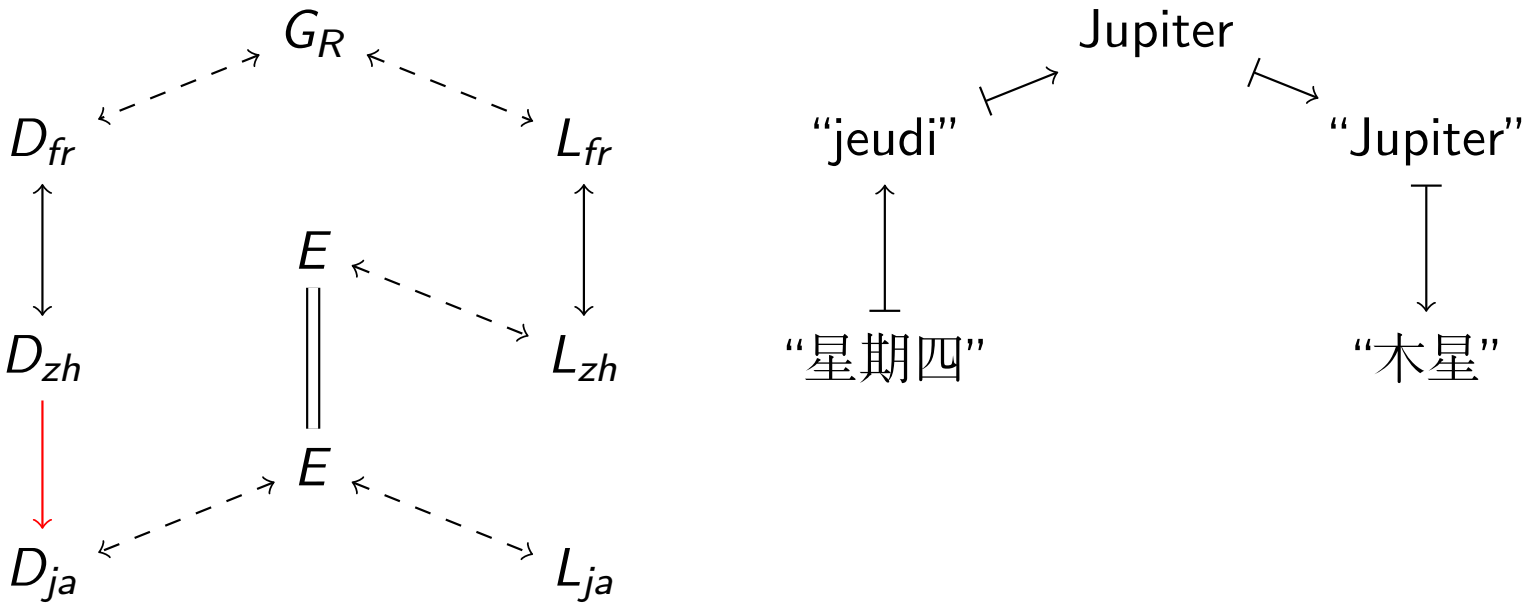

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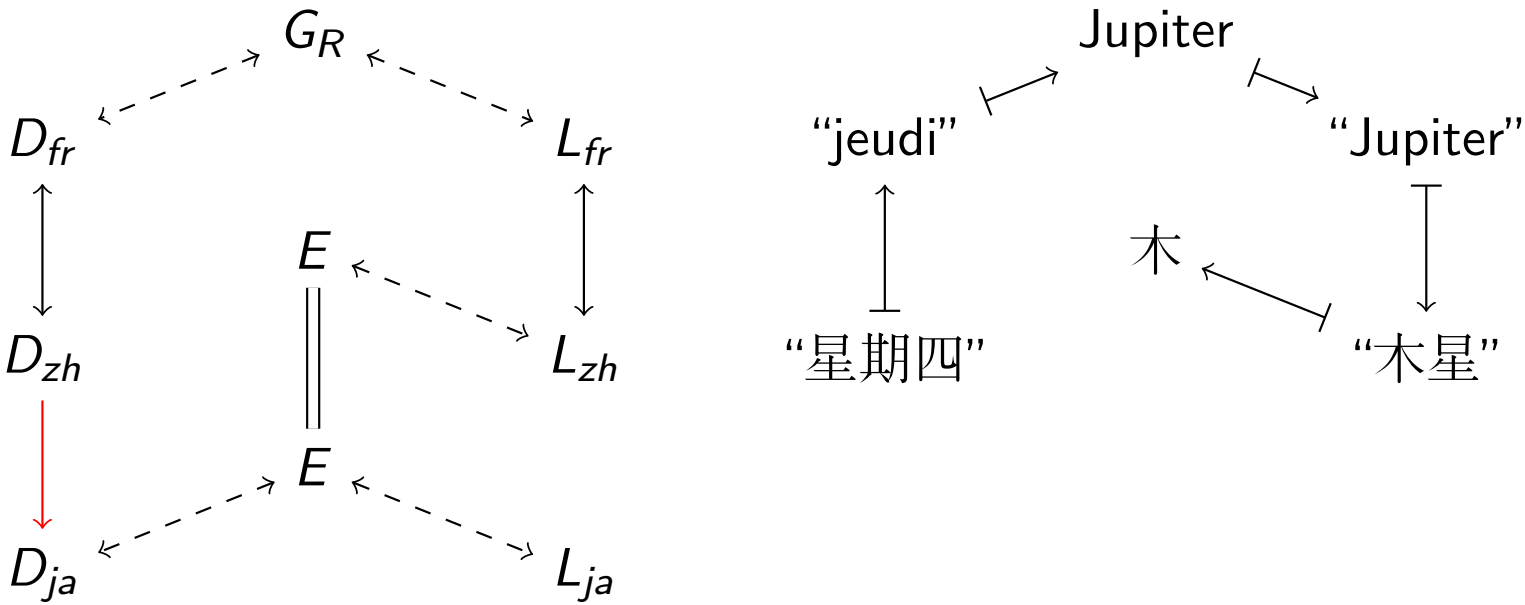

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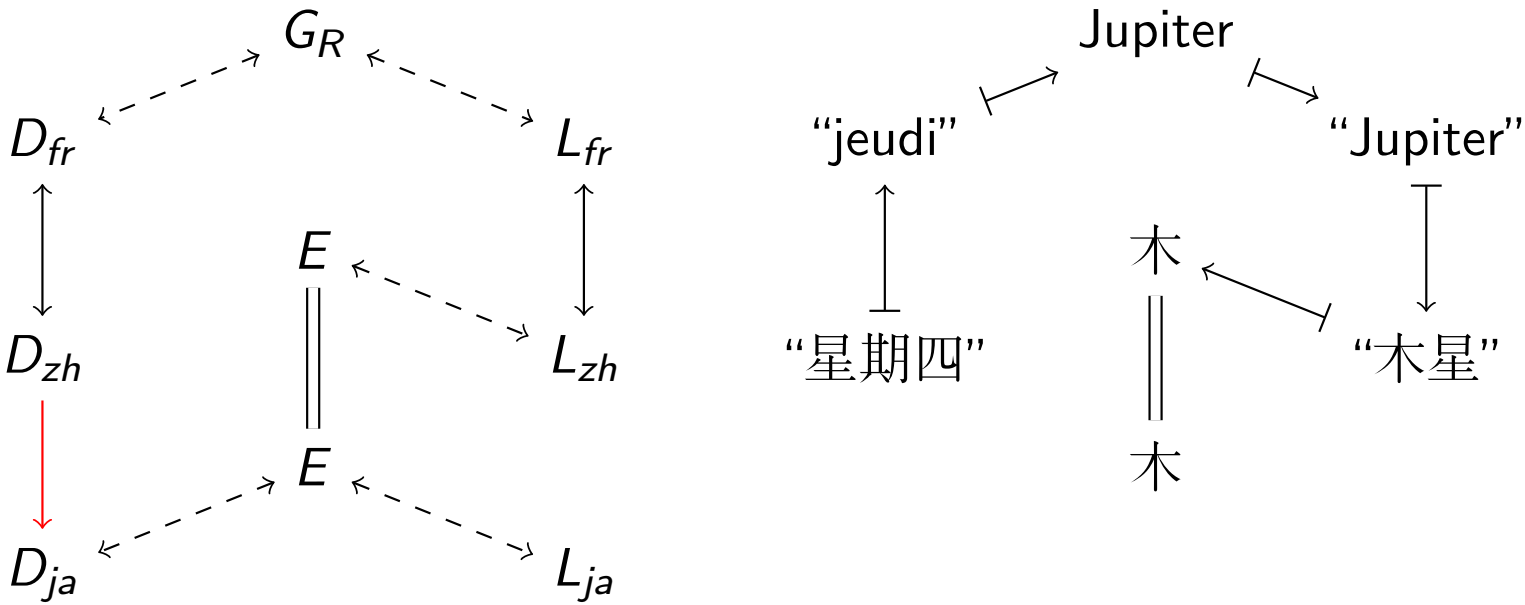

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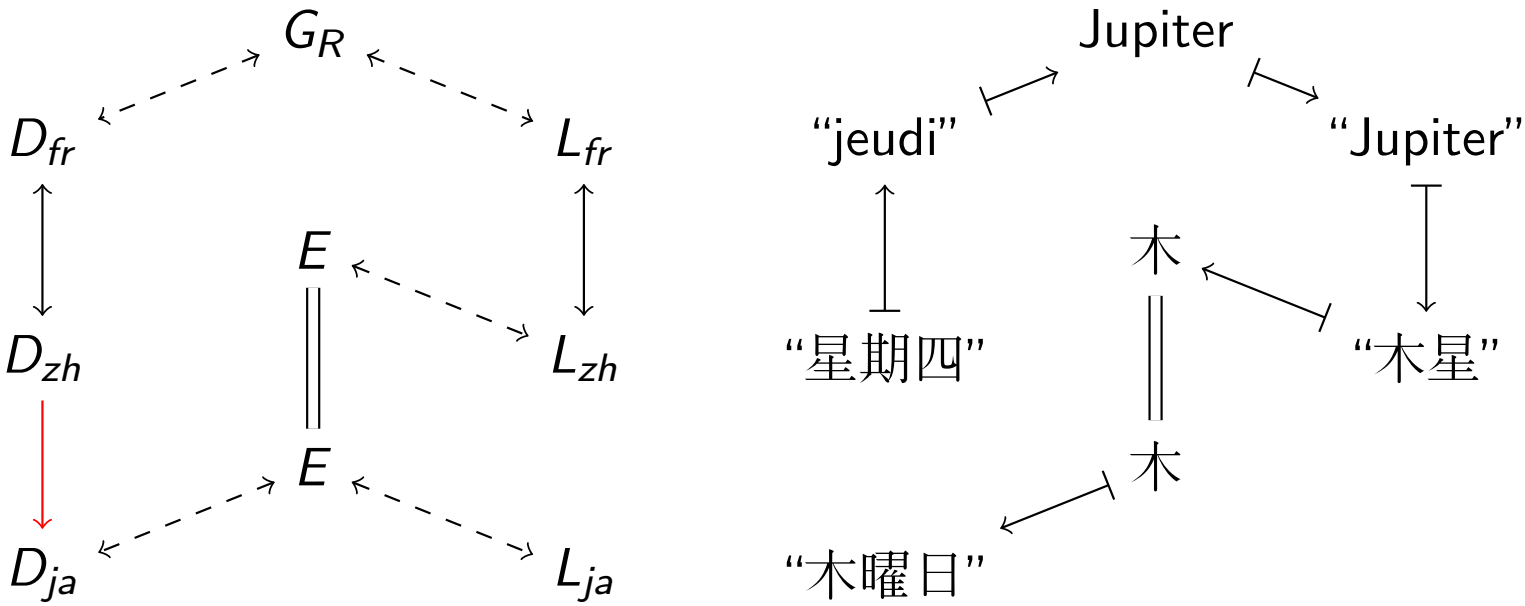
- “木星”(Wood star), 木(Wood)

Example: How do you say 星期四(Star period 4) in Japanese?



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- Answer: “木曜日”(Wood luminary day)

Goal 2: Order on L

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We want to relate the order on L induced from $D \longleftrightarrow L$

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$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
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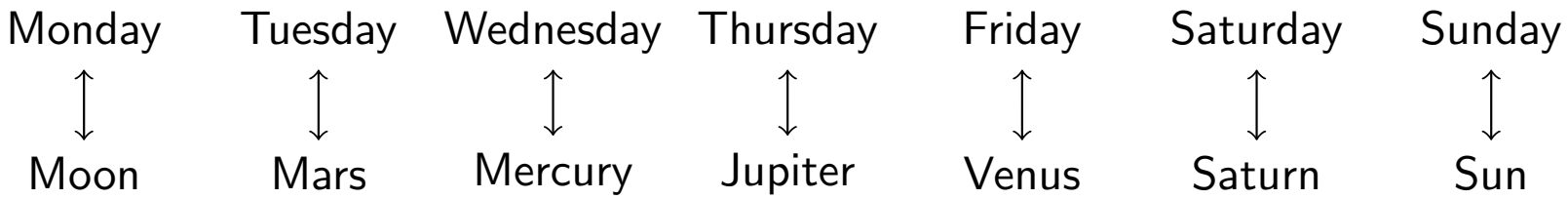

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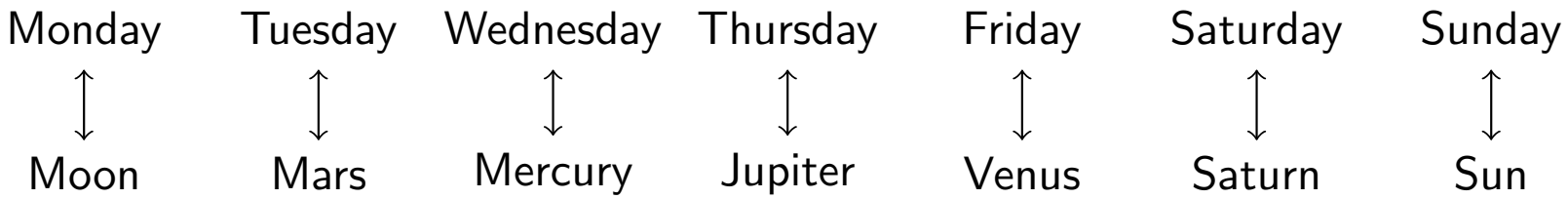

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Reverse-engineering the planetary day system

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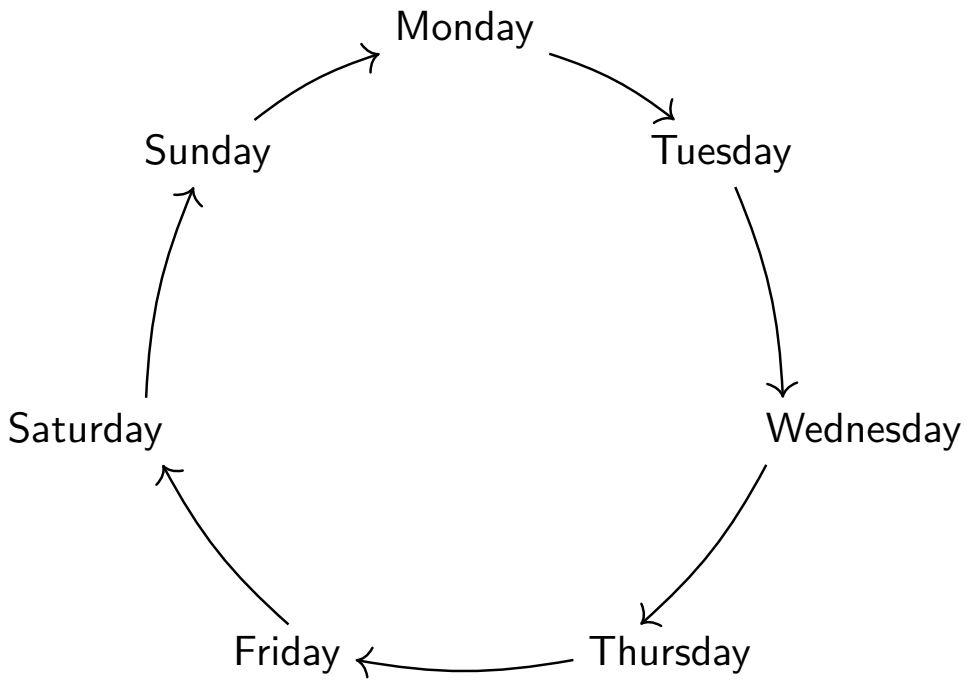

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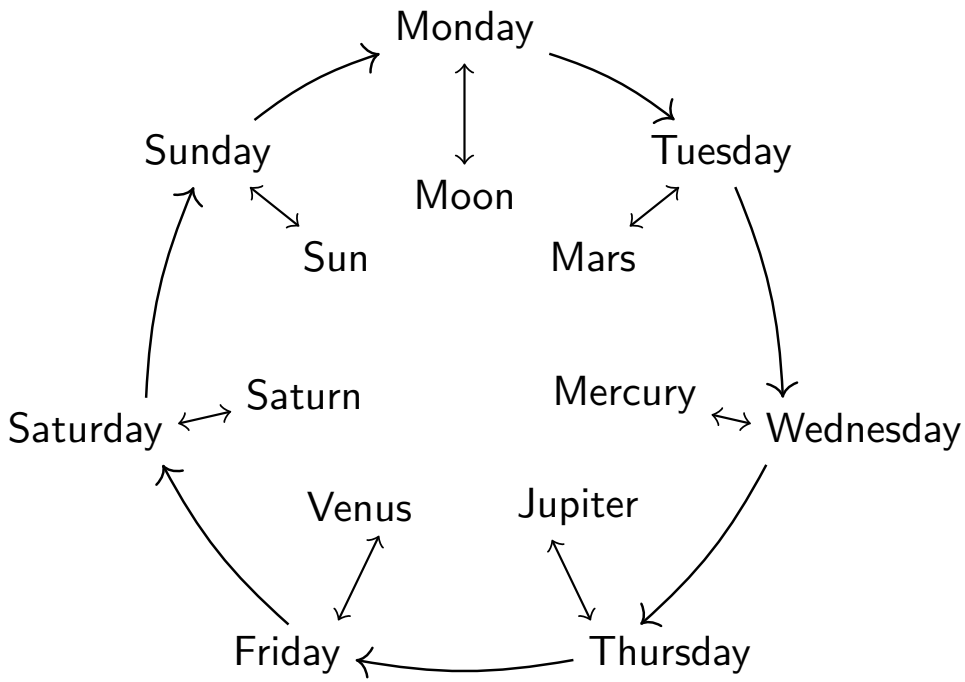

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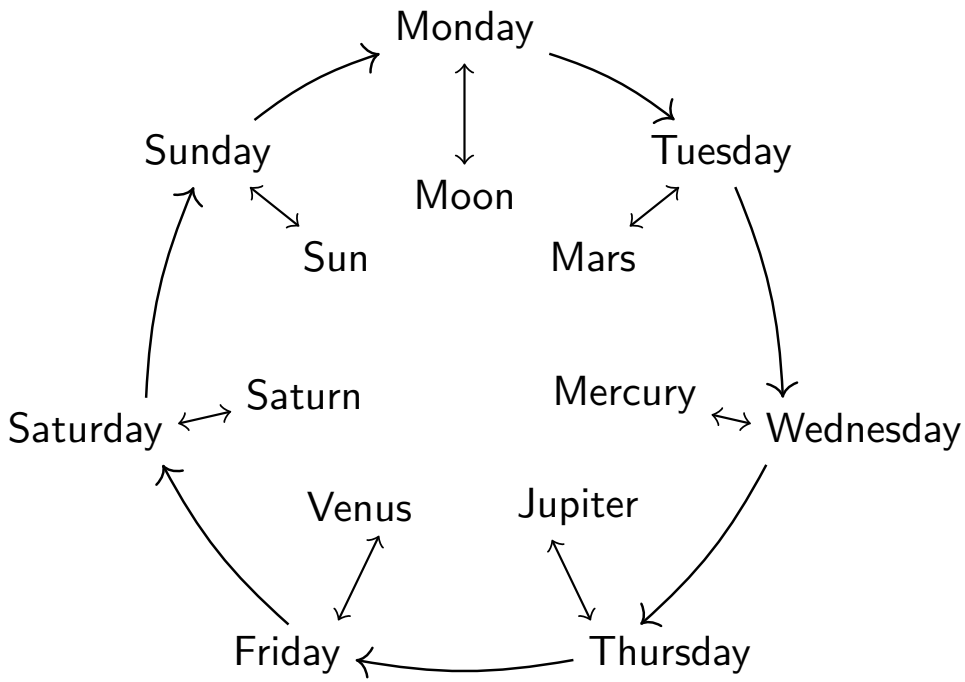

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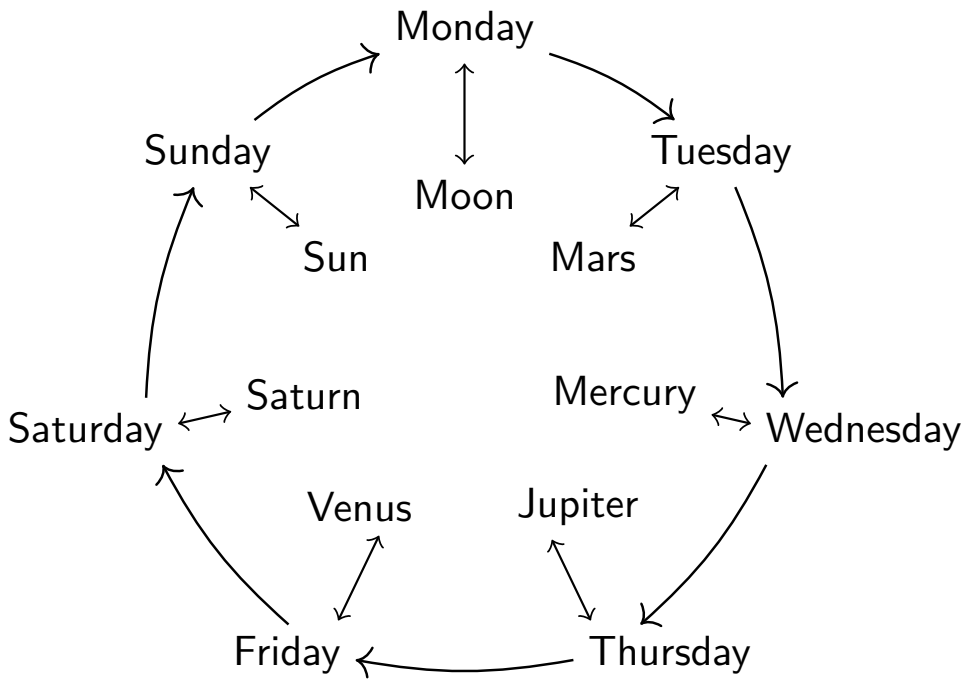

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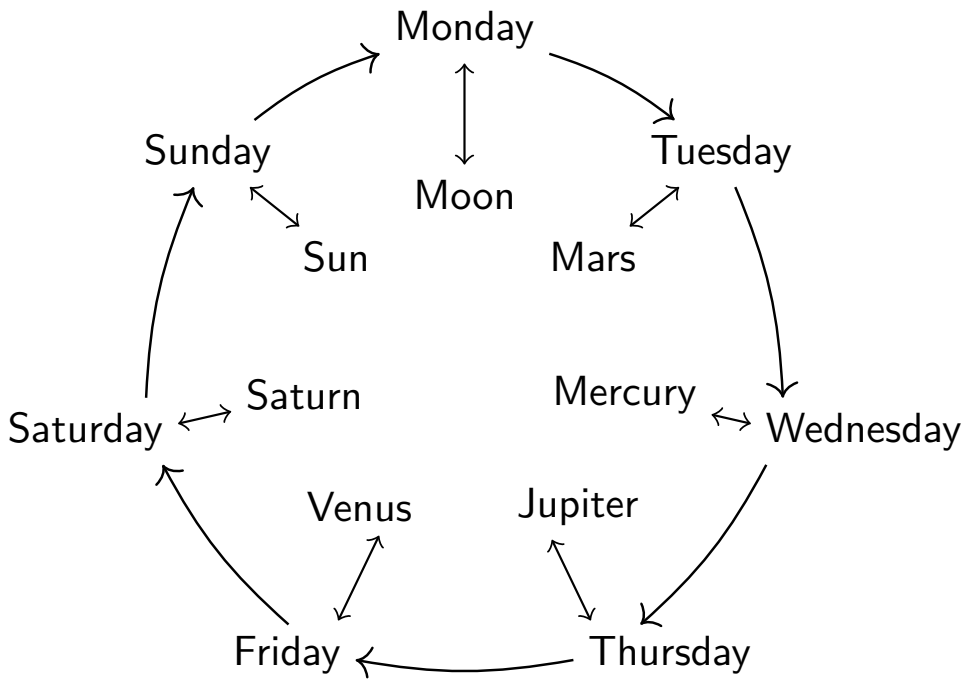

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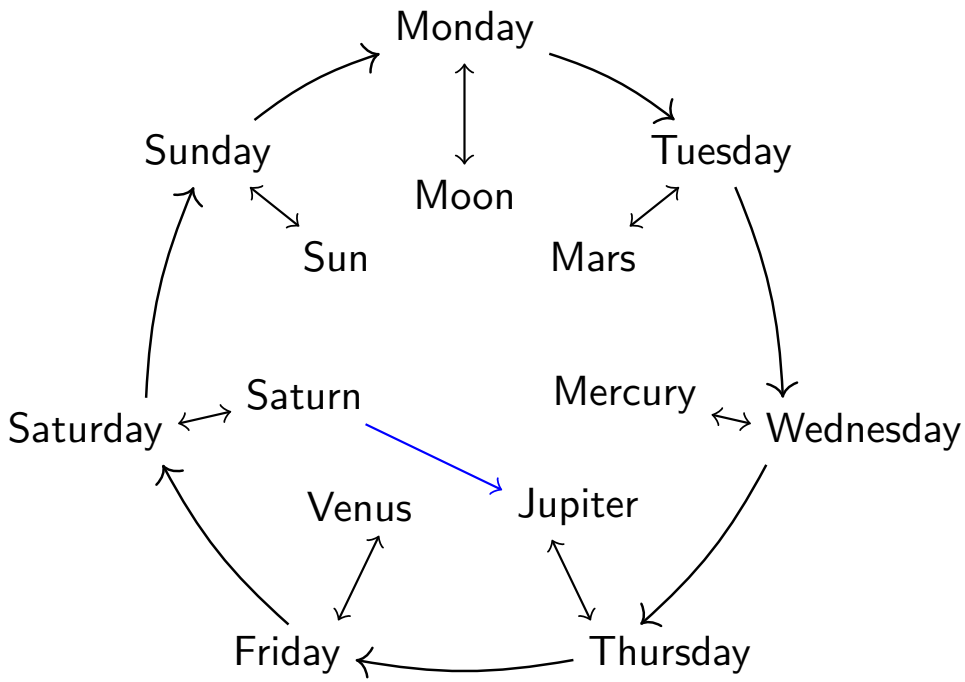

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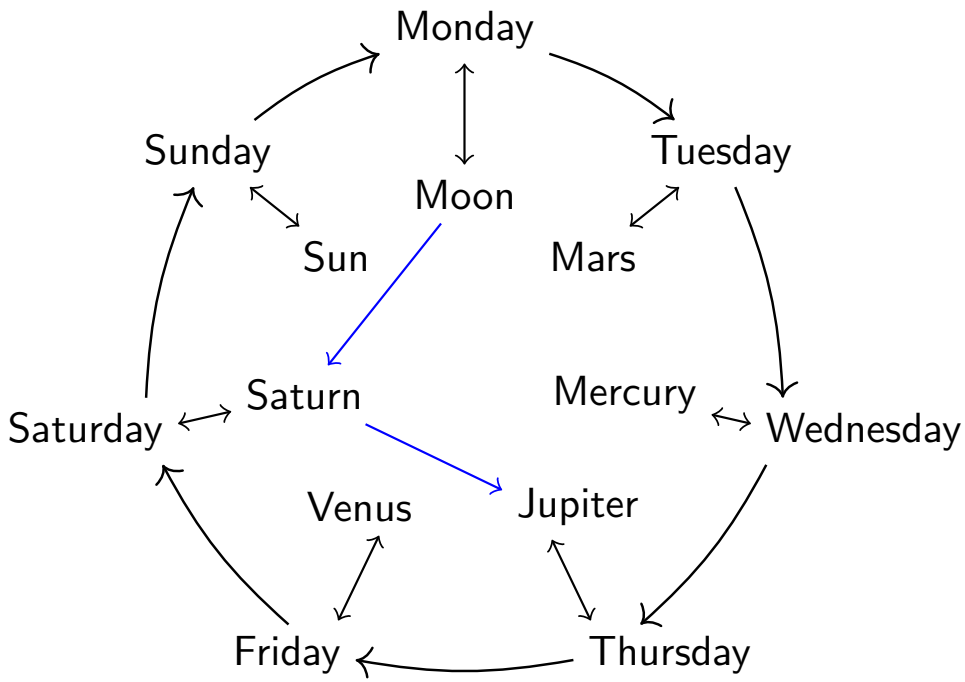

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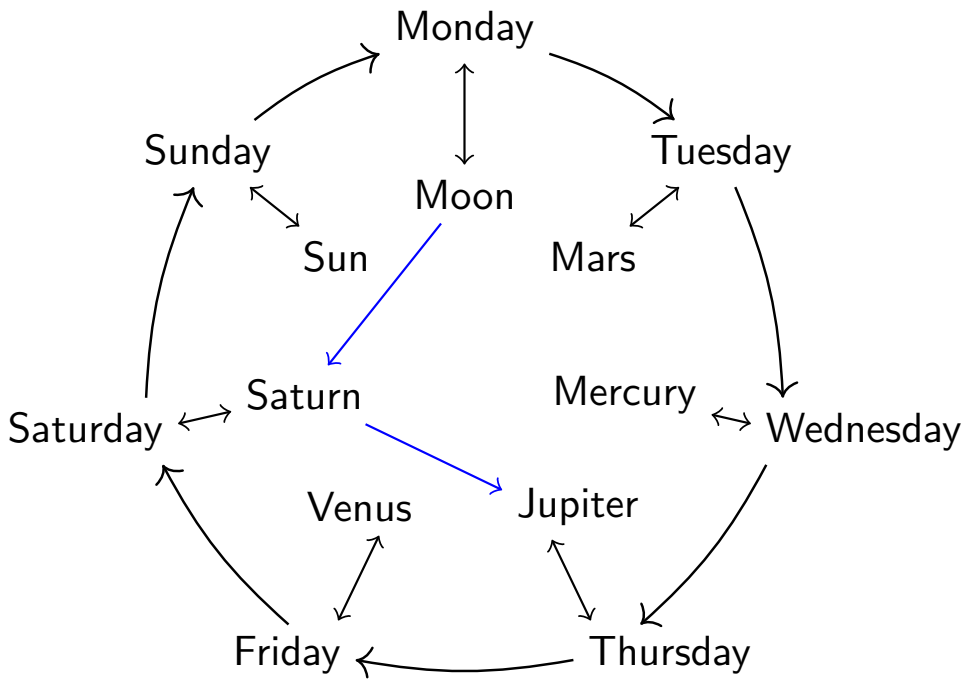

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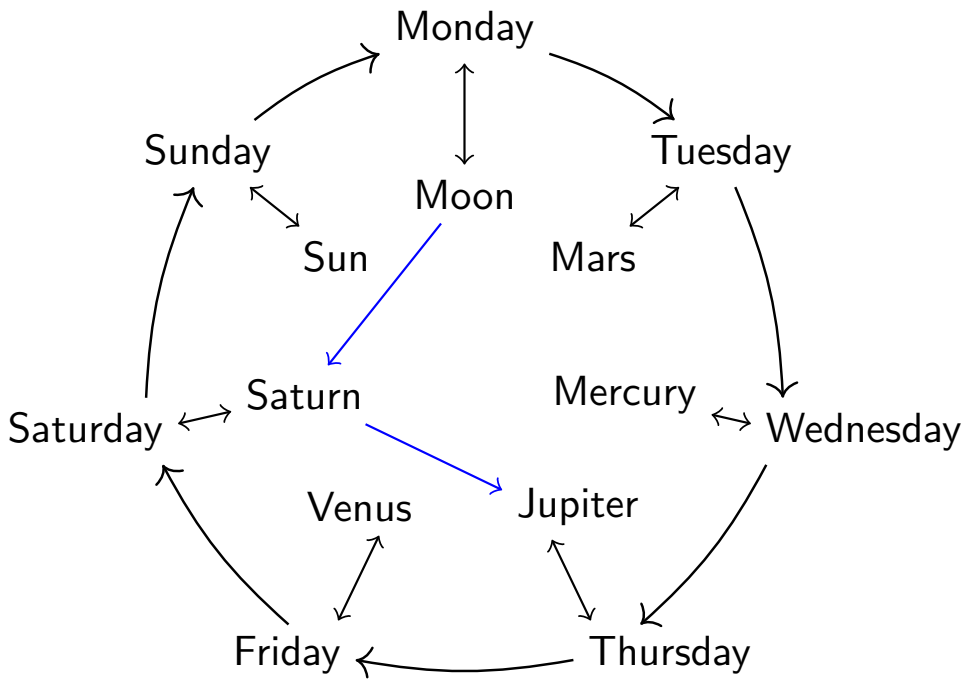

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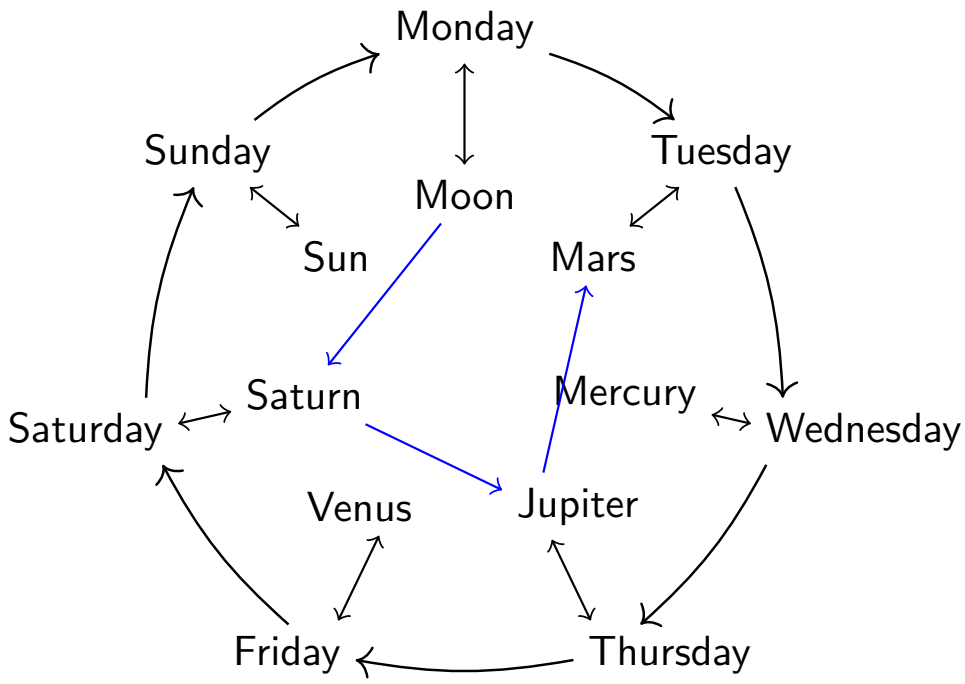

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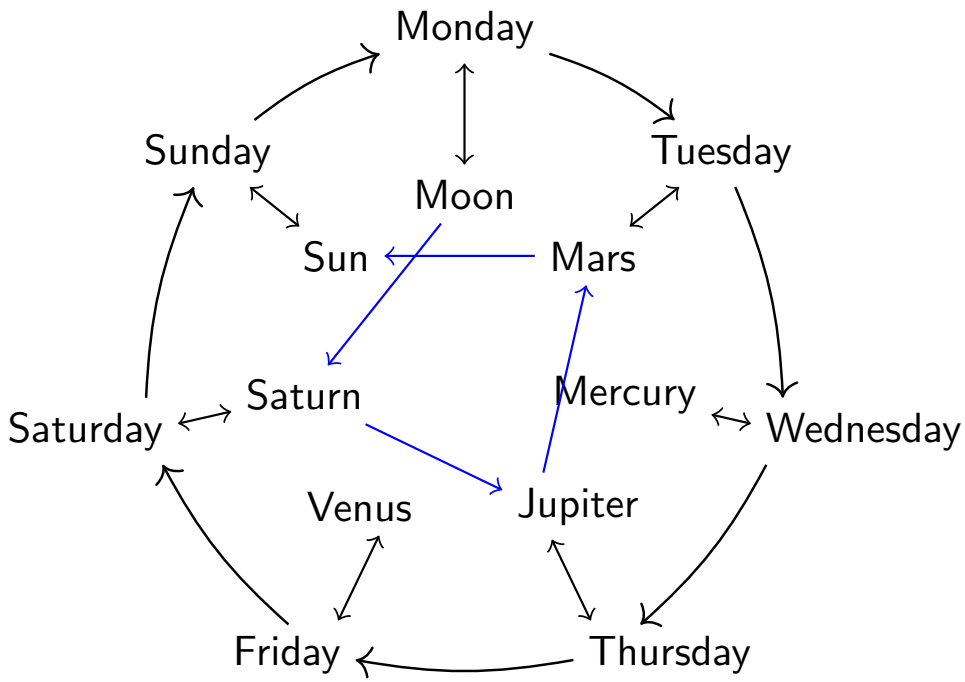

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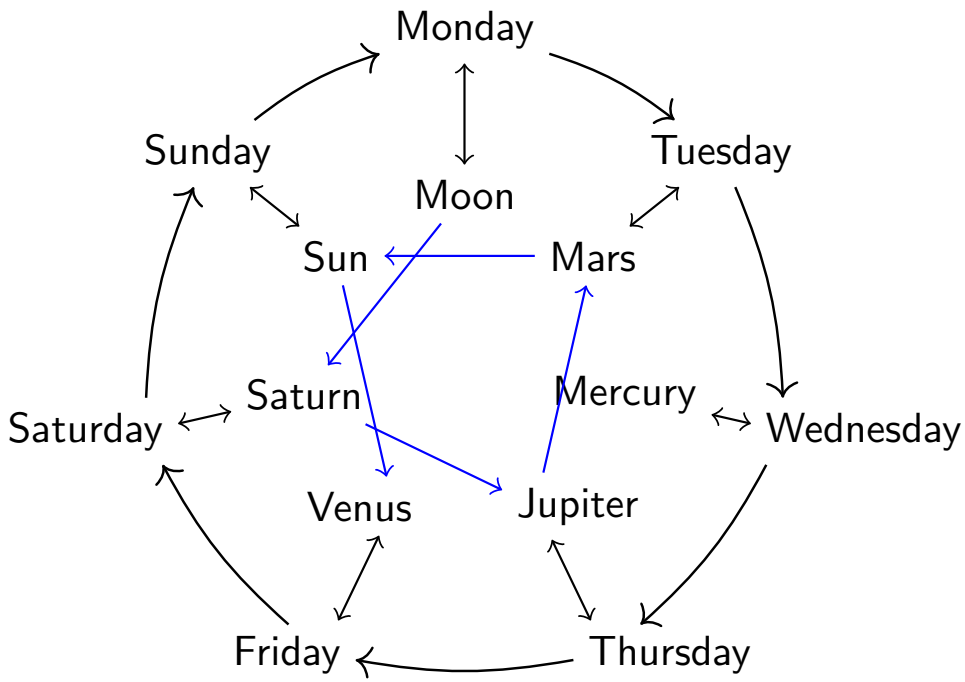

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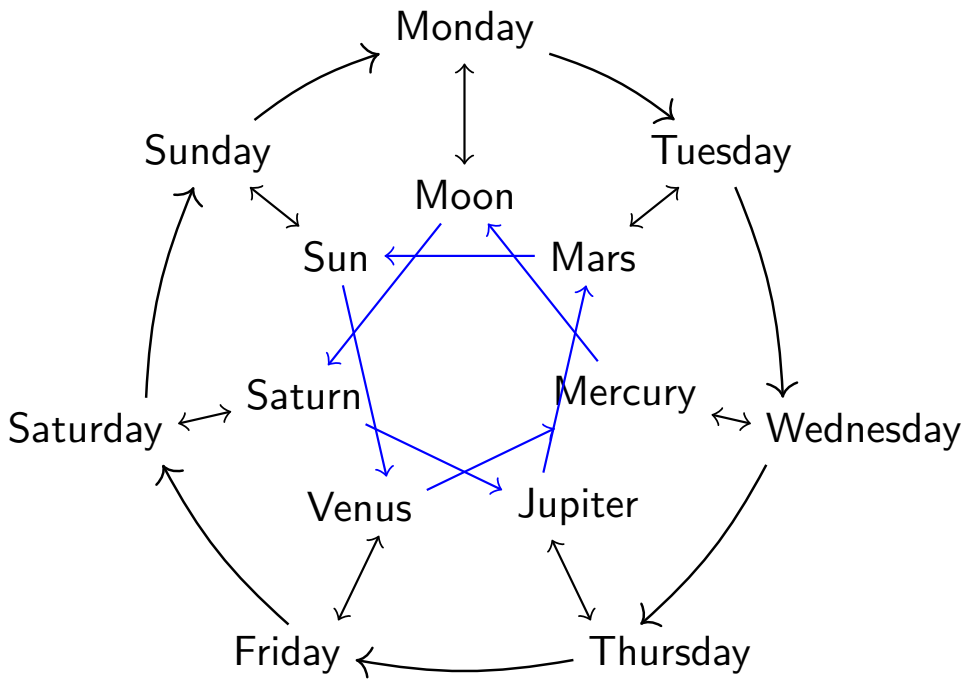

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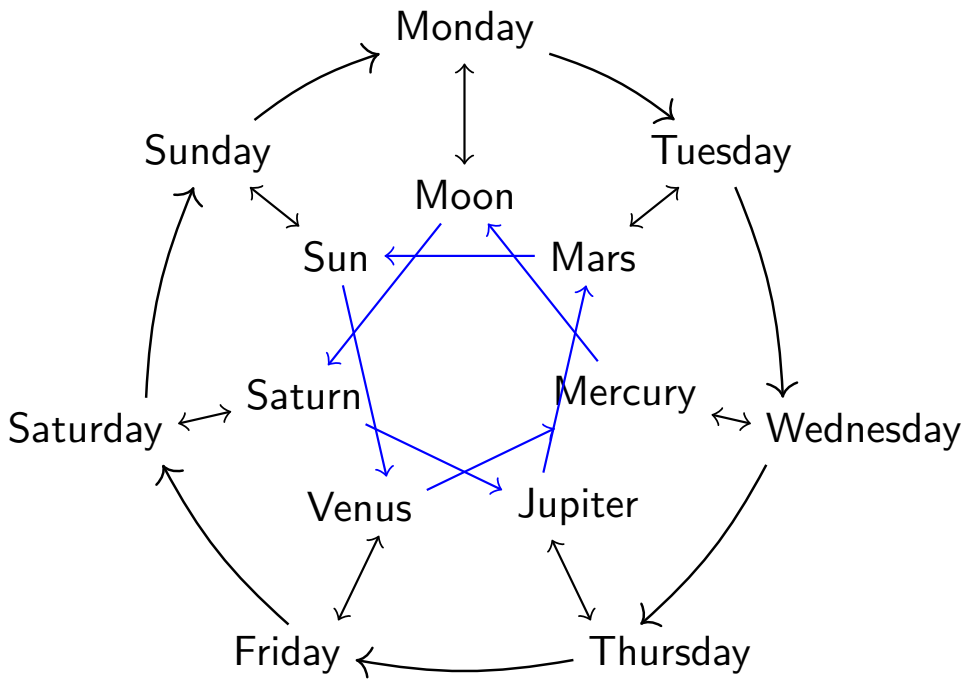

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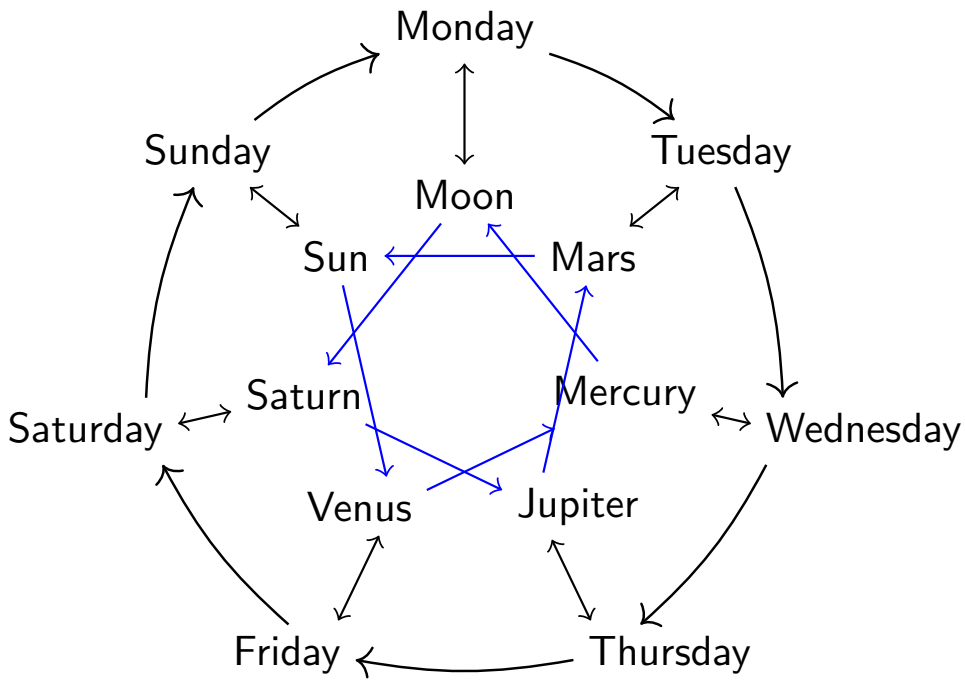

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- This is in fact e.g. Ptolemy's order

Figure : The Celestial Spheres (Peter Apian, *Cosmographia*, Antwerp, 1524)

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	1	2	3	$\cdots$	24
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$$3 \equiv 5^{-1} \bmod 7 \quad (\text{i.e. } 3 \cdot 5 \equiv 1 \bmod 7)$$

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- **Answer:** Almost, but not quite
- Need to make precise the **structures involved** and what the planetary hour system says about them

Relevant structure: Cyclically ordered sets

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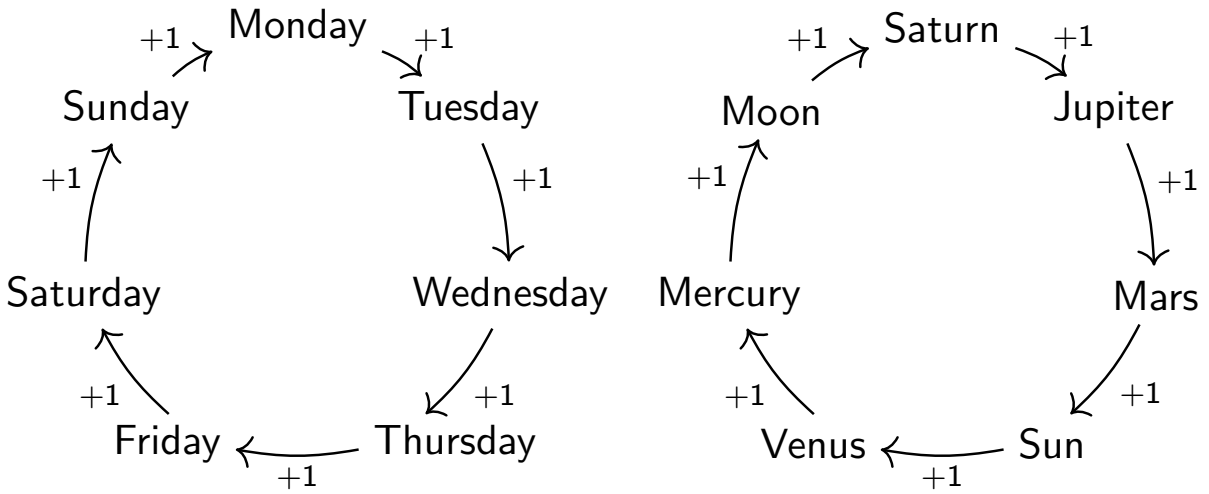

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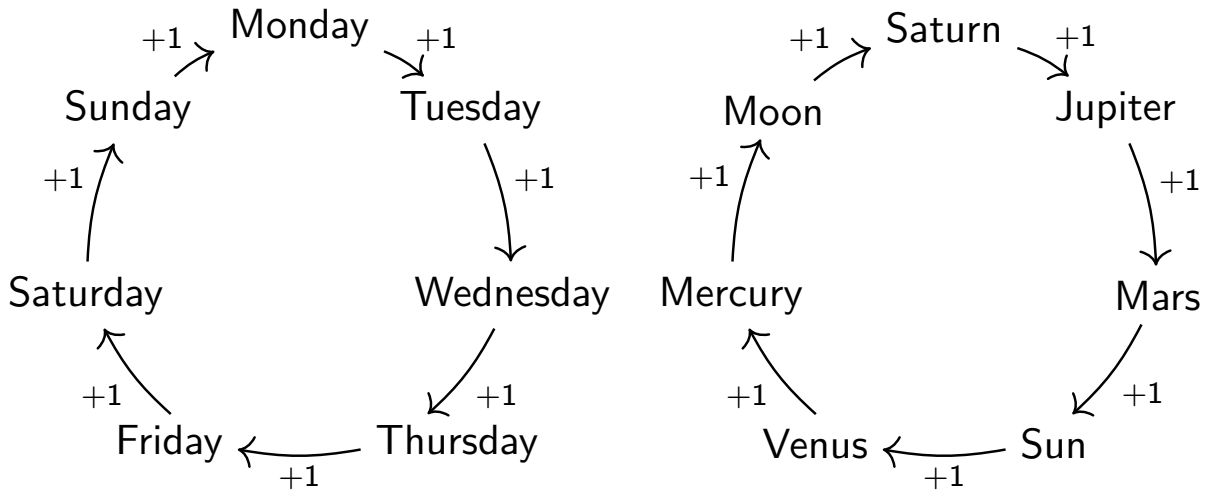

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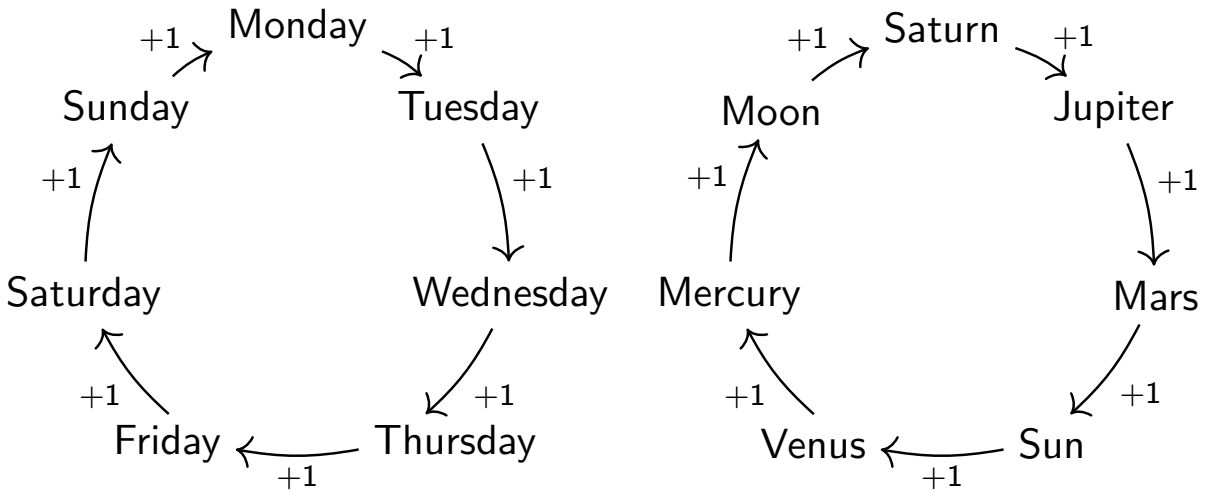

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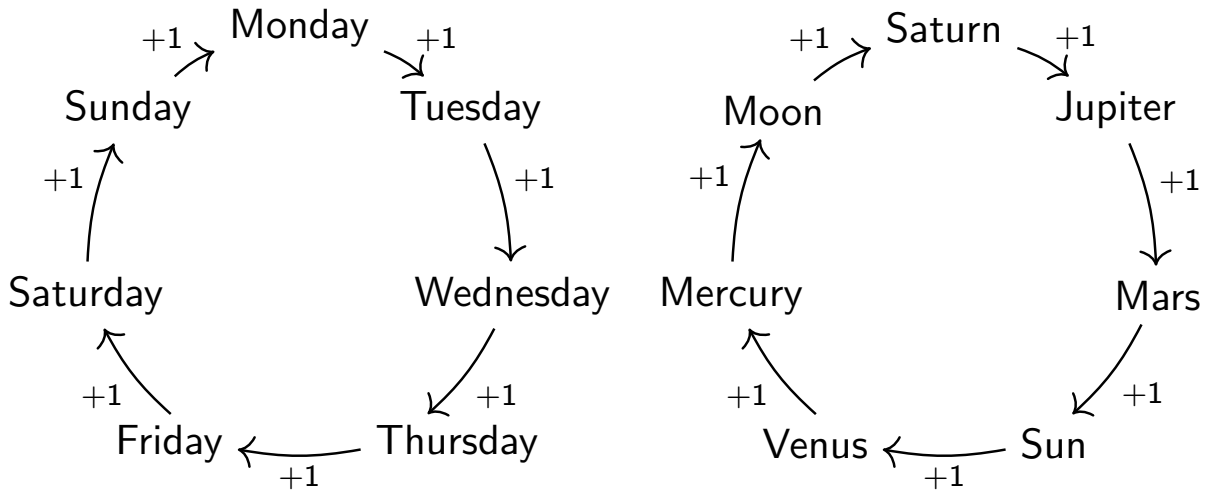

- $+1 : D \longrightarrow D$ sends each day to the next
- $+1 : L \longrightarrow L$ sends each luminary to the one one closer to the Earth (according to the ancients)
- May as well consider $(D, +1)$ and $(L, +3)$

Planetary hour system

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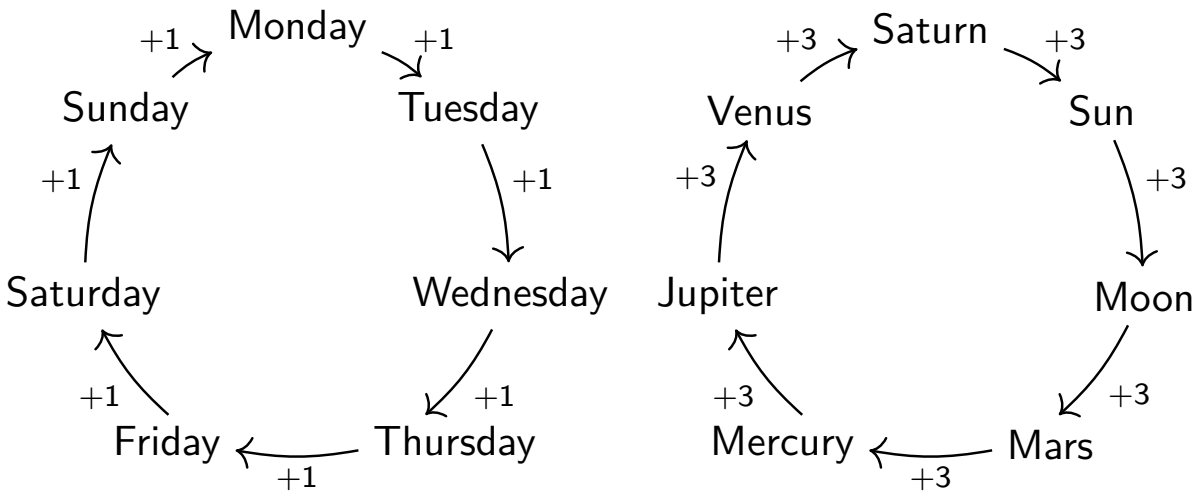

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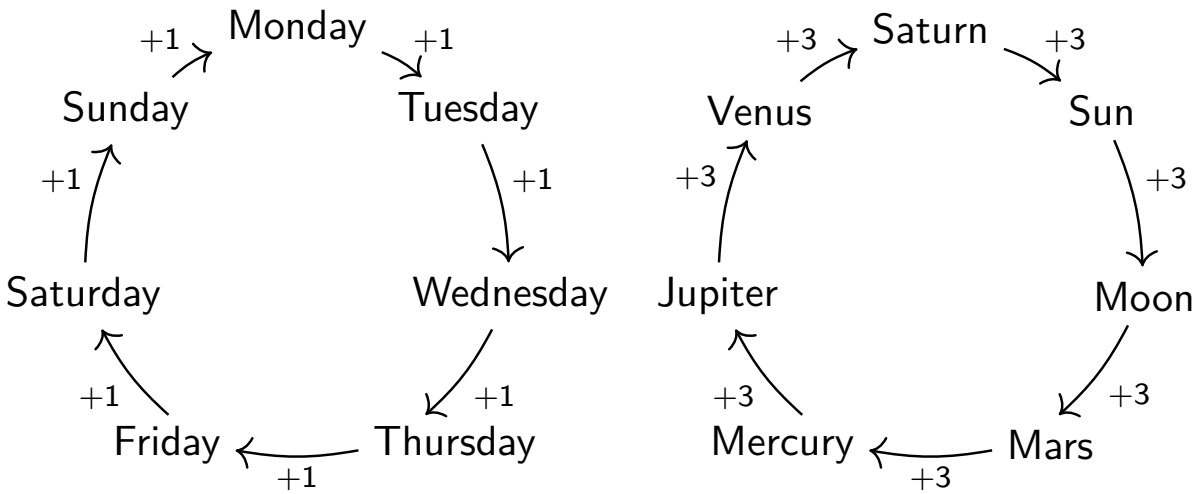

The planetary hour system is a correspondence $(D, +1) \longleftrightarrow (L, +3)$ of cyclically ordered sets.

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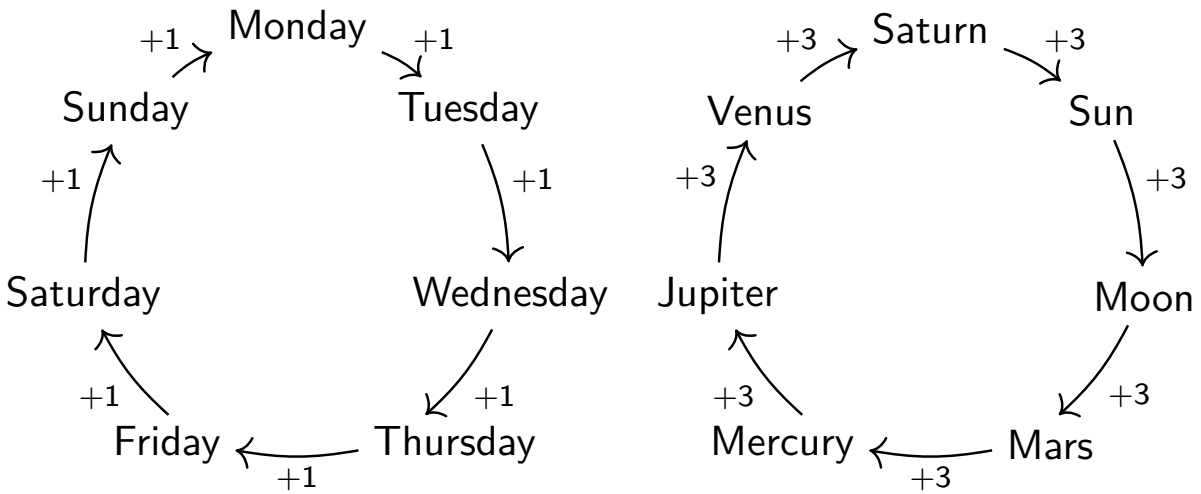

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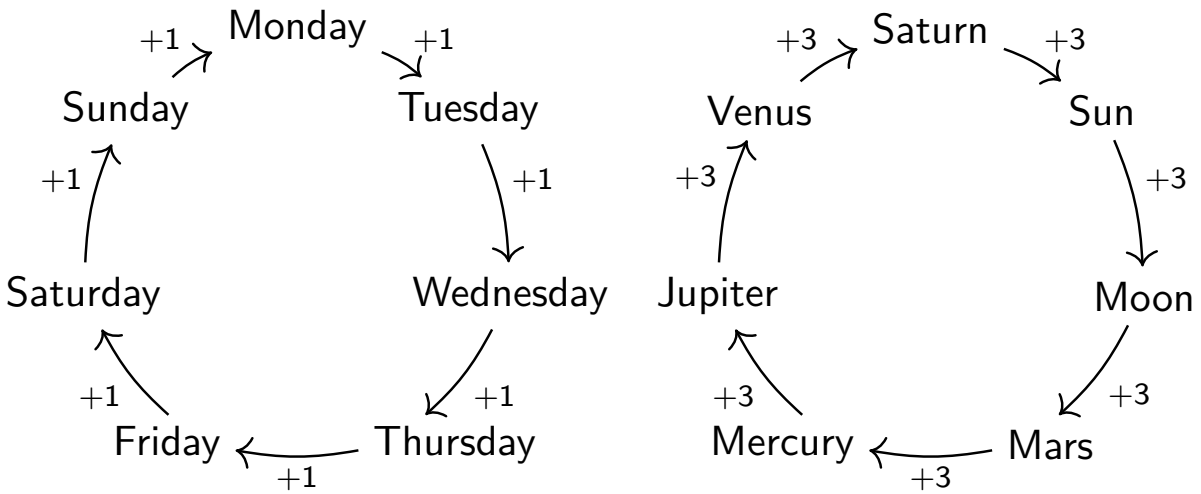

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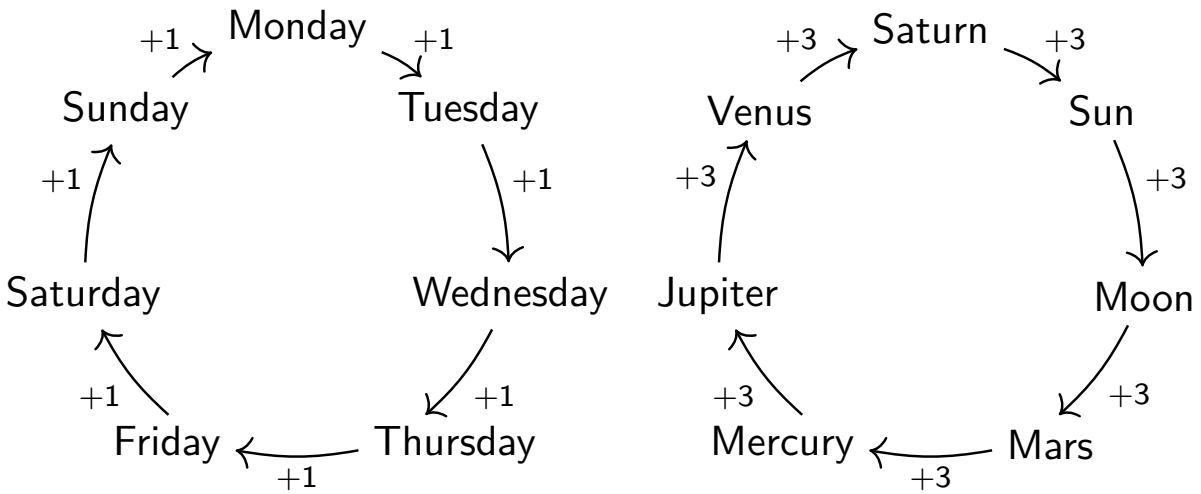

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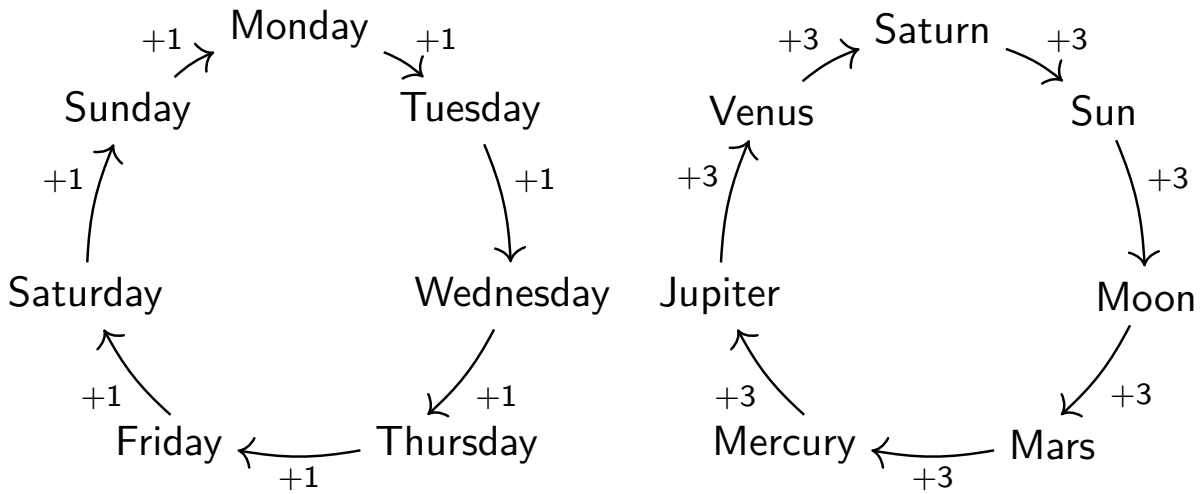

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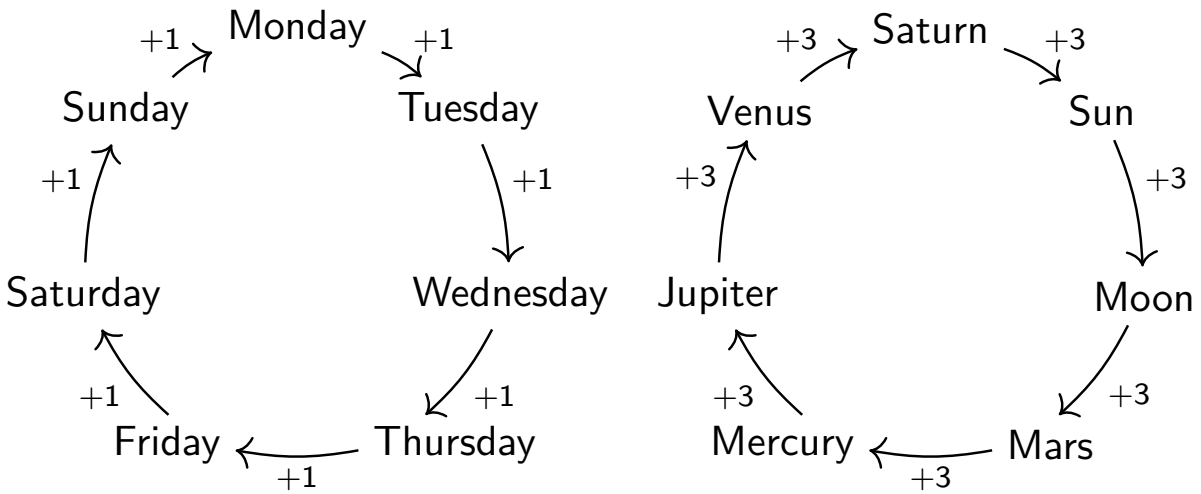

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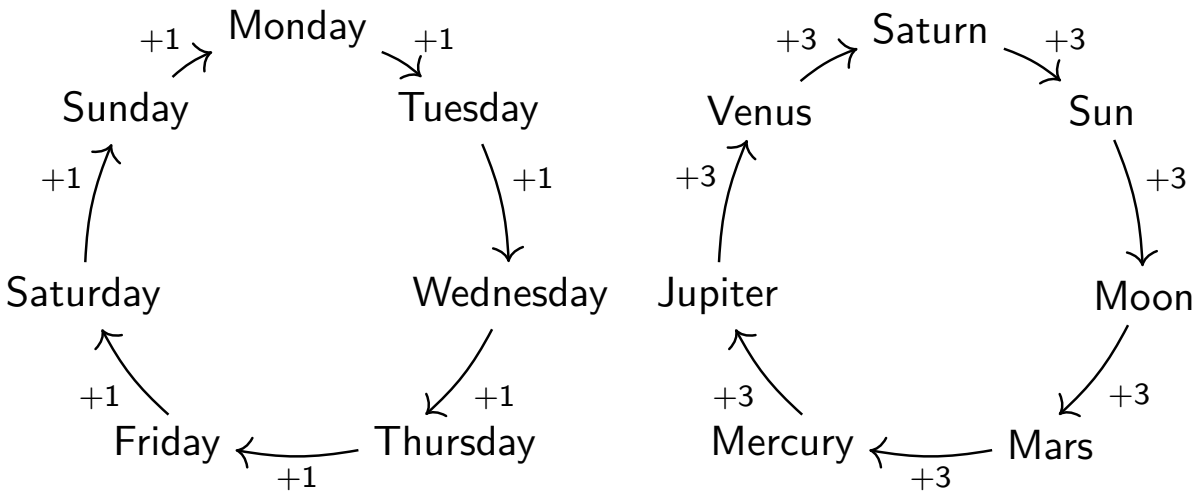

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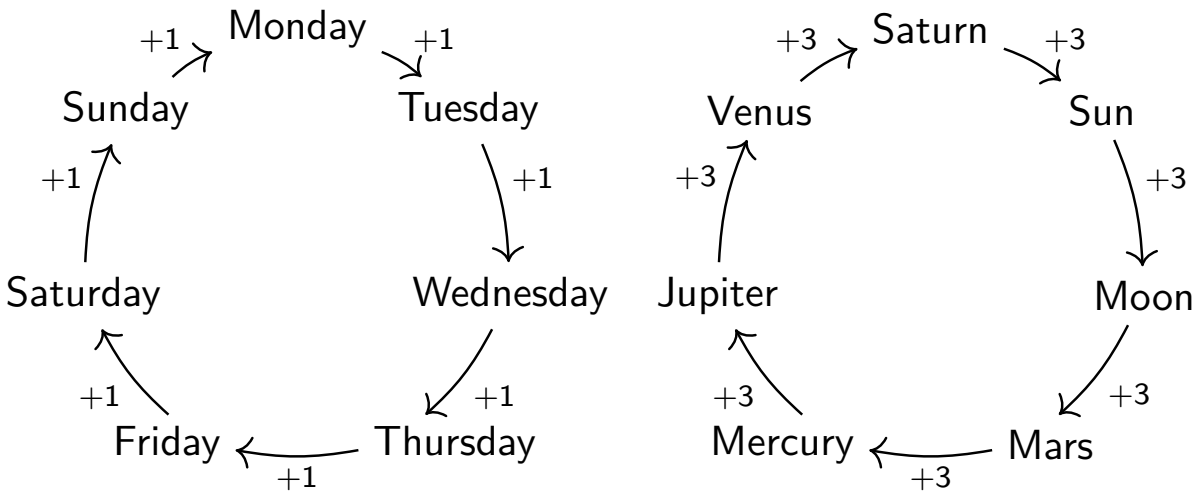

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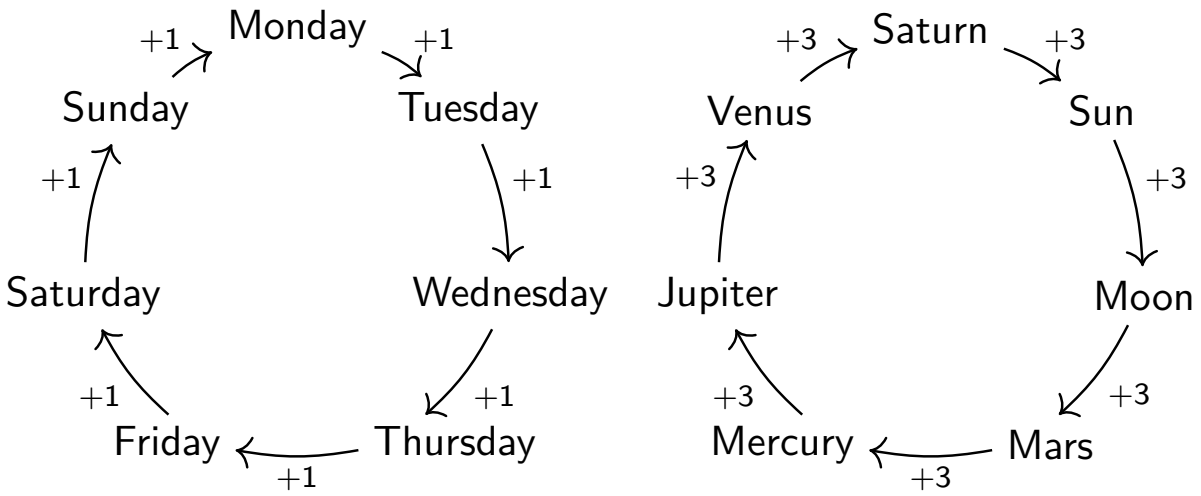
- There are 7 such correspondences

Planetary hour system



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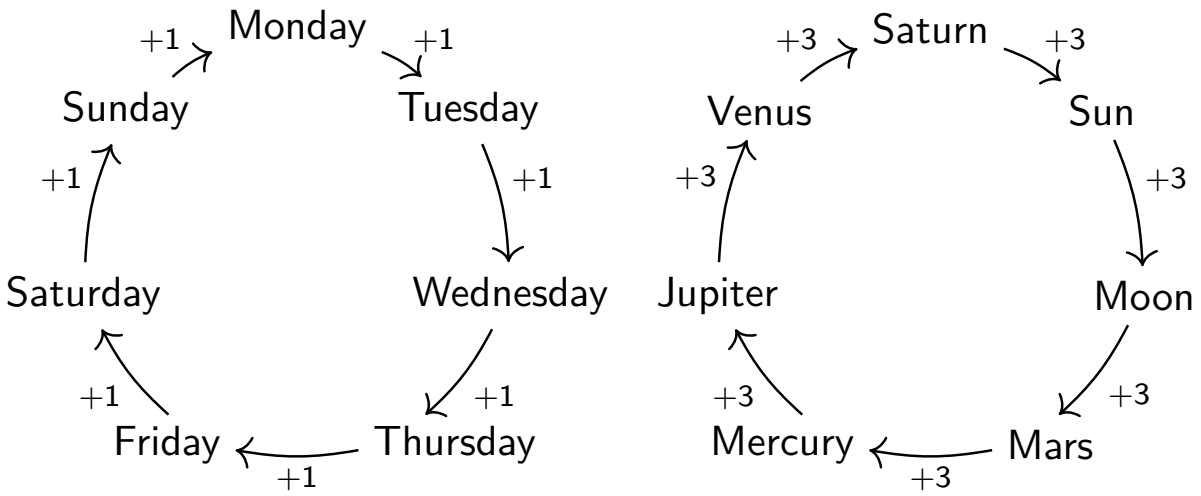

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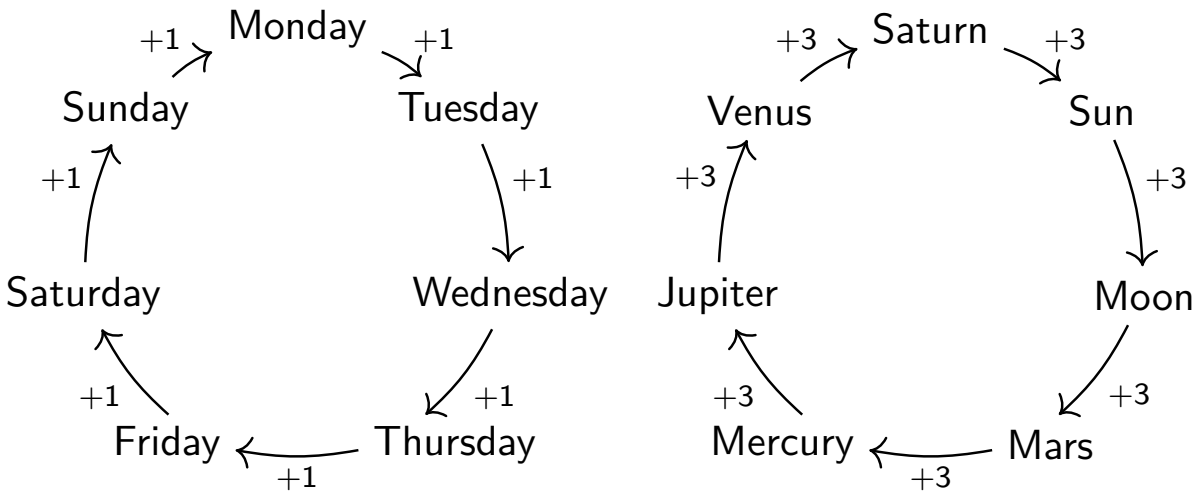

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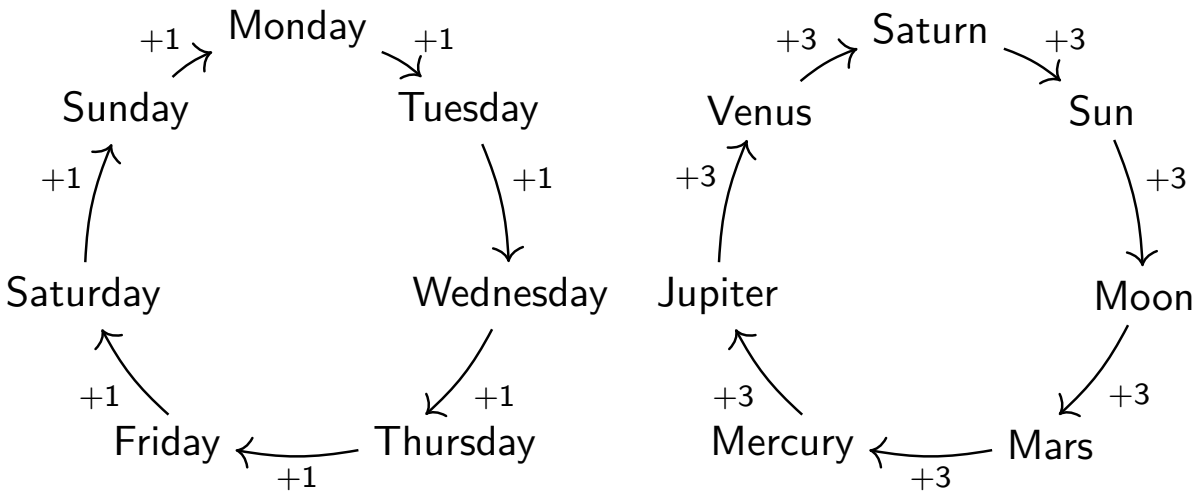

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  - Chinese has exactly one: 星期日(Star period Day) $\longleftrightarrow$ 太阳(Sun)

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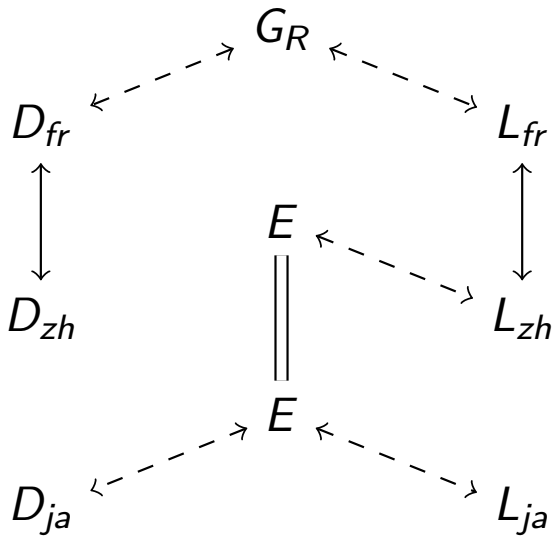

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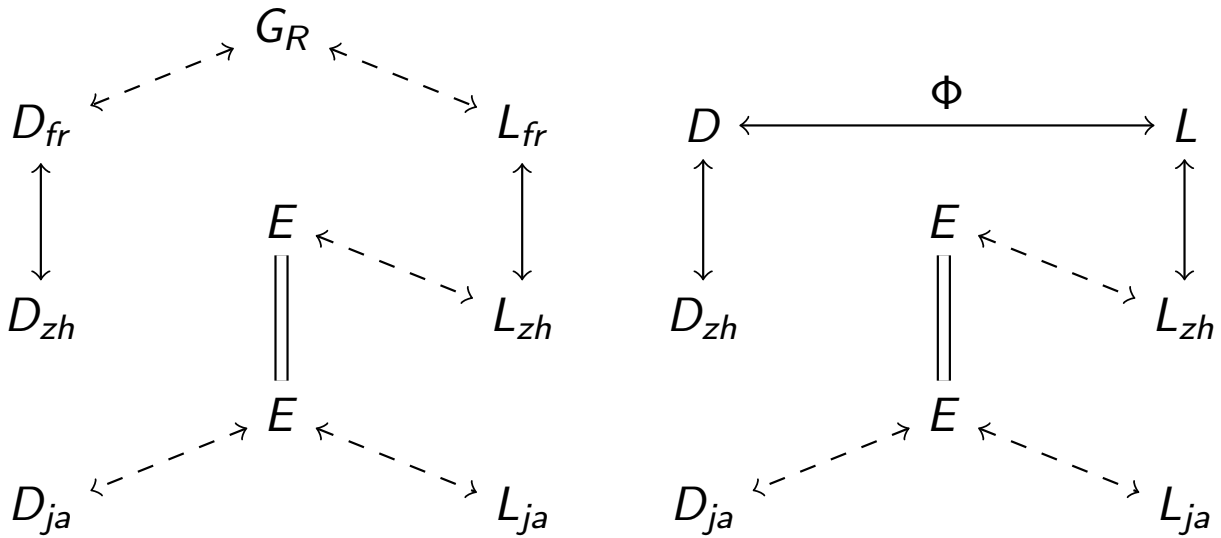

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so look at the following commutative diagram given by the planetary hour system:

$$\begin{array}{ccc} \text{星期日(Star period Day)} & \xleftrightarrow{\Phi} & \text{太阳(Sun)} \\ \Big\downarrow{\scriptstyle (+1)^4} & & \Big\downarrow{\scriptstyle (+3)^4} \\ \text{星期四(Star period 4)} & \xleftrightarrow[\Phi]{} & ? \end{array}$$

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As before, using the element 木(Wood), the answer is 木曜日(Wood luminary day)

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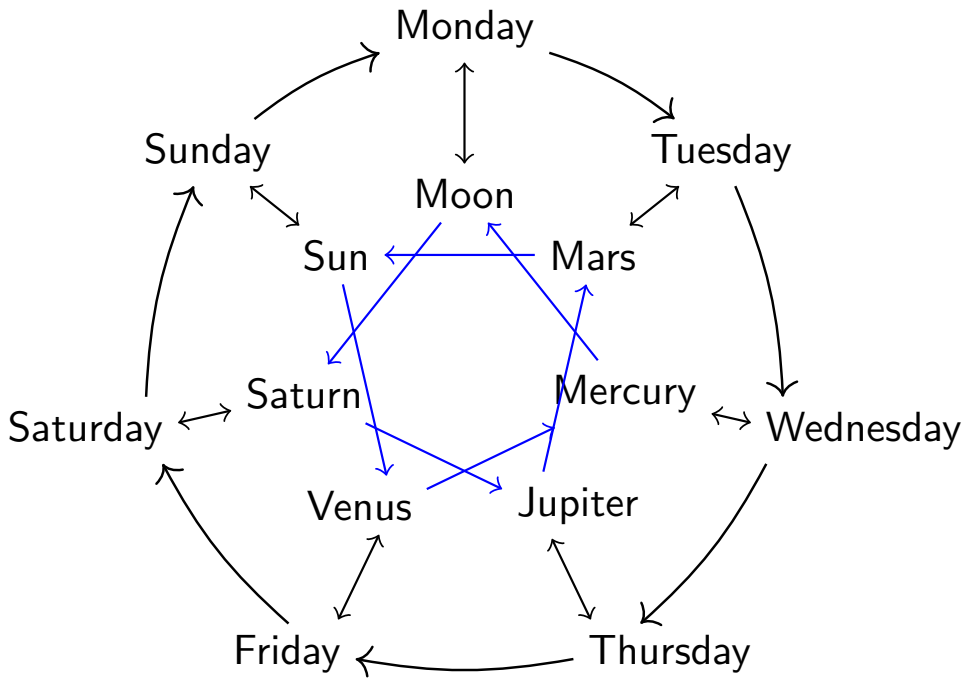
Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday
Moon
Sun
Mars
Mercury
Venus
Jupiter
Saturn

Thank you