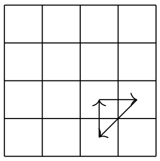
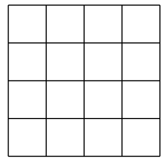
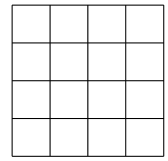
HOW TO SOLVE THE 15-PUZZLE BLINDFOLDED: EXERCISES

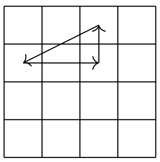
1. Notation for 3-cycles

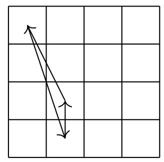
Problem 1: Understanding 3-cycle notation. Fill in the blank.

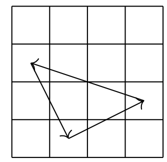
(11 12 15) =  ,

(12 15 11) =  ,

(10 2 8) =  ,

(__ __ __) =  ,

(__ __ __) =  ,

(__ __ __) =  .

Problem 2: Applying 3-cycles. Fill in the blank.

1	5	8	10
11	2	6	9
14	12	3	7
4	15	13	

(11 12 15) =

1	5	8	10
11	2	6	9
14	12	13	3
4	15	7	

,

1	5	8	10
11	2	6	9
14	12	3	7
4	15	13	

(10 2 8) =

,

7	12	11	14
10	3	15	13
2	6	9	8
5	4	1	

(3 5 7) =

,

7	12	11	14
10	3	15	13
2	6	9	8
5	4	1	

(1 14 10) =

.

Note that e.g. (11 12 15) doesn't have to move the tiles 11, 12, 15!

Problem 3: Identifying 3-cycle needed to solve a state. Consider the following puzzle states:

1.

1	2	3	4
5	6	7	8
9	10	12	15
13	14	11	

2.

8	2	3	4
5	6	7	10
9	1	11	12
13	14	15	

3.

1	8	3	4
5	6	7	12
9	10	11	2
13	14	15	

4.

1	2	3	4
5	6	13	8
9	10	11	12
14	7	15	

For each state, write down the 3-cycle you need to apply to turn it into the solved state.

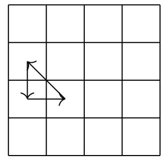
1. (11 12 15) 2. (__ __ __) 3. (__ __ __) 4. (__ __ __)

Date: April 22, 2017.

2. Basic 3-cycles

In the problems below, assume that position 16 is blank.

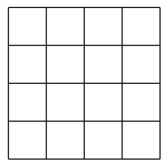
1. Fill in the blanks to explain how to perform the 3-cycle

(5 9 10) = 

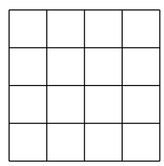
in three steps:

Step 1: Make position __ blank by doing the following moves: ______________
Step 2: Use this blank to perform the 3-cycle in 4 moves: ______________
Step 3: Reverse the moves from Step 1: ______________

2. Put this into practice! Record your current puzzle state:

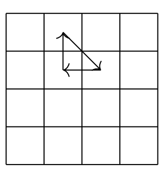
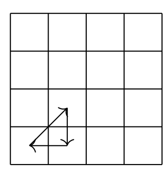
(your current state) =  .

Compute what should happen if you apply (5 9 10) to your current state:

(your current state)(5 9 10) =  .

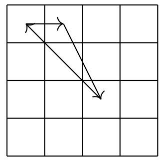
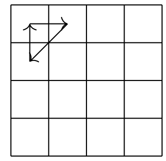
Perform the moves you found in the previous part. Did you get what you expected?

3. Repeat this for the following 3-cycles:

(2 7 6) =  , (14 13 10) = 

3. Easy 3-cycles

1. Consider the following 3-cycles:

$(1\ 2\ 11) =$  , $(1\ 2\ 5) =$  .

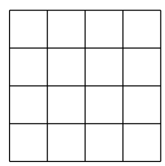
We want to reduce the 3-cycle $(1\ 2\ 11)$ to the basic 3-cycle $(1\ 2\ 5)$ using conjugation:

$$(1\ 2\ 11) = x(1\ 2\ 5)x^{-1}$$

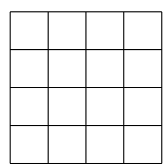
for some sequence of moves x. For this to work, x should:
- move the tile in position __ to position __;
- make position __ blank;
- without affecting positions __ and __.

Find such a sequence of moves x. (Hint: First, apply D so that position 12 becomes blank.)

2. Put this into practice! Record your current state:

your current state =  .

Compute what should happen if you apply $(1\ 2\ 11)$ to your current state:

(your current state)$(1\ 2\ 11) =$  .

Using the sequence of moves x you found, apply $x(1\ 2\ 5)x^{-1}$ to your current state. Did you get what you expected?

3. Repeat this for other 3-cycles: $(10\ 11\ 13)$, $(1\ 3\ 4)$, $(3\ 5\ 6)$, make up your own!

4. Hard 3-cycles

1. Use the rule

$$(a\ b\ c) = (a\ b\ d)(a\ d\ c).$$

to rewrite the following 3-cycles as a product of two easy 3-cycles:

$$(2\ 8\ 10) = (\underline{\quad}\ \underline{\quad}\ \underline{\quad})(\underline{\quad}\ \underline{\quad}\ \underline{\quad}).$$

2. Reduce each easy 3-cycle to a basic 3-cycle using conjugation:

$$(\underline{\quad}\ \underline{\quad}\ \underline{\quad}) = x(\underline{\quad}\ \underline{\quad}\ \underline{\quad})x^{-1}, \quad \text{where} \quad x = \underline{\hspace{6cm}},$$
$$(\underline{\quad}\ \underline{\quad}\ \underline{\quad}) = y(\underline{\quad}\ \underline{\quad}\ \underline{\quad})y^{-1}, \quad \text{where} \quad y = \underline{\hspace{6cm}}.$$

3. Put this into practice! Record your current state:

your current state =

.

Compute what should happen if you apply (2 8 10) to your current state:

(your current state)(2 8 10) =

.

Using the sequences you found above, apply (2 8 10) to your current state. Did you get what you expected?

4. Repeat this for other 3-cycles: (5 14 3), (13 11 2), make your own!

5. Cycle decomposition and simplification

In these problems, you will work out the cycle decomposition and simplification for the following puzzle states:

1.

7	11	15	14
4	12	13	1
3	8	9	10
5	2	6	

2.

10	14	8	13
2	3	1	6
9	5	4	12
11	15	7	

3.

5	15	4	1
12	6	13	14
7	3	8	2
9	10	11	

4.

4	12	1	3
10	15	13	14
6	7	5	8
9	2	11	

5.

12	10	2	7
14	6	1	13
3	9	8	11
15	4	5	

6.

7	3	11	15
12	4	10	14
5	9	13	1
8	2	6	

7.

1	7	14	13
10	8	6	12
9	5	3	15
11	4	2	

8.

15	14	13	12
11	10	7	8
9	6	5	4
3	2	1	

Problem 1: Cycle decomposition. For each state, write down the permutation you need to solve it as a product of disjoint cycles.

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____

Problem 2: Cycle simplification. Recall the following cycle simplification rules:

$$(a\ b\ c\ d) = (a\ b\ c)(a\ d)$$
$$(a\ b\ c\ d\ e) = (a\ b\ c)(a\ d\ e)$$
$$(a\ b\ c\ d\ e\ f) = (a\ b\ c)(a\ d\ e)(a\ f)$$
$$\vdots$$

For each state, use these rules to express the permutation you found in Problem 1 as a product of 3-cycles and 2-cycles.

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________
7. ______________________________
8. ______________________________

Problem 3: Pairs of 2-cycles. Recall the following rule:

$$(a\ b)(c\ d) = (a\ b\ c)(a\ d\ c).$$

For each state, express the permutation you found in Problem 1 as a product of 3-cycles.

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________
7. ______________________________
8. ______________________________